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the lectures pdfs are available at:

https://www.physics.umd.edu/rgroups/amo/orozco/results/2022/Results22.htm

Correlations in Optics and Quantum Optics; A series of lectures about correlations and coherence. November 2022

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BOS.QT

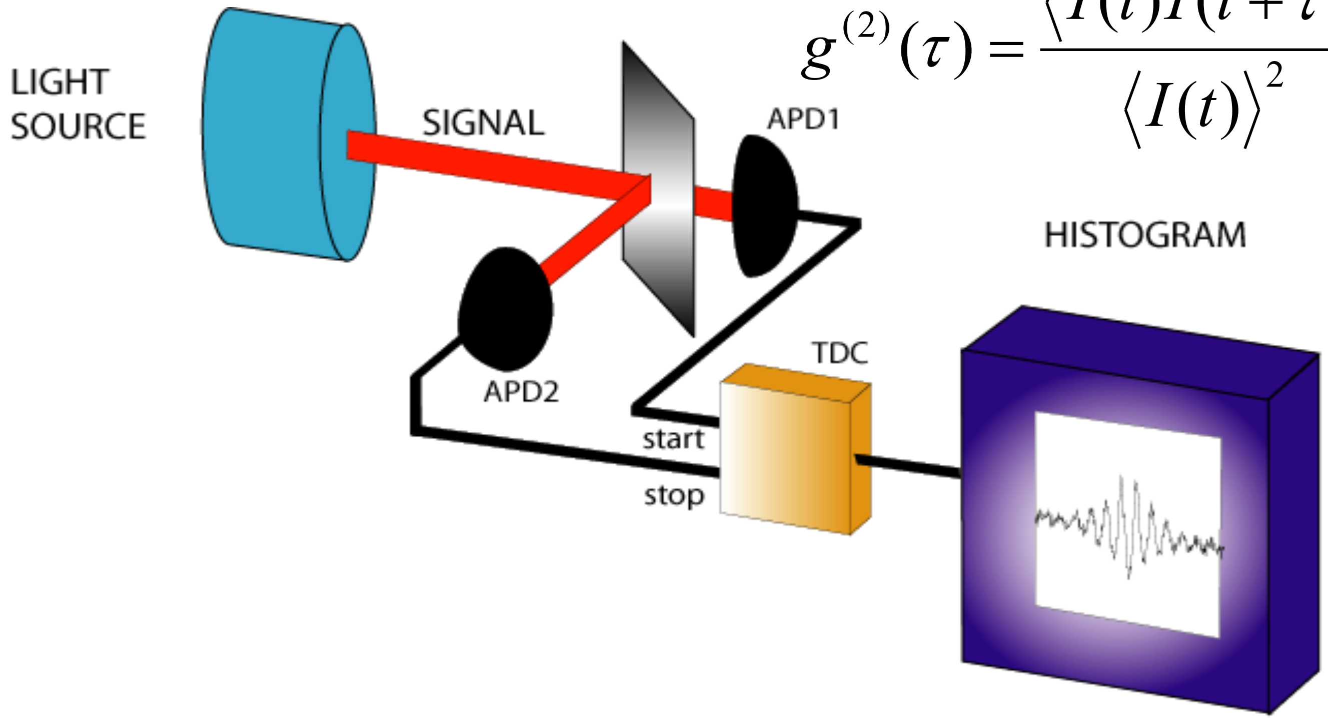
Lesson 7

Tentative list of topics to cover:

- From statistics and linear algebra to power spectral densities
- Historical perspectives and examples in many areas of physics
- Correlation functions in classical optics (field-field; intensity-intensity; field-intensity) part iii
- Optical Cavity QED
- Correlation functions, quantum examples
- Correlations of the field and intensity
- Correlations and conditional dynamics for control
- From Cavity QED to waveguide QED.

Hanbury Brown and Twiss; Intensity Intensity correlation

$$g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle^2}$$



R. Hanbury Brown and R.Q. Twiss, Correlation between Photons in Two Coherent Beams of Light, Nature 177, 27

Correlations of the intensity at τ=0

$$g^{(2)}(0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2}$$

$$= \frac{\langle (I_0 + \delta(t))^2 \rangle}{\langle I_0 + \delta(t) \rangle^2}$$

$$= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

It is proportional to the variance

Intensity correlations (bounds)

$$g^{(2)}(0) = 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

$$g^{(2)}(0) - 1 \geq 0$$

Cauchy-Schwarz

$$2I(t)I(t+\tau) \leq I^2(t) + I^2(t+\tau)$$

$$|g^{(2)}(\tau) - 1| \leq |g^{(2)}(0) - 1|$$

The correlation is maximal at equal times (τ=0) and it can not increase.

Quantum Correlations (Glauber):

$$g^{(2)}(\tau) = \frac{\langle T : \hat{I}(t)\hat{I}(t+\tau) : \rangle}{\langle \hat{I}(t) \rangle^2}$$

The intensity operator *I* is proportional to the number of photons, but the operators have to be normal (:) and time (*T*) ordered. All the creation operators do the left and the annihilation operators to the right (just as a photodetector works). The operators act in temporal order.

R. Glauber, *"The Quantum Theory of Optical Coherence,"* Phys. Rev. **130**, 2529 (1963).

At equal times (normal order) :

$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2}.$$

Conmutator : $\hat{a}^{+}\hat{a} = \hat{a}\,\hat{a}^{+} - 1$

$$\left\langle \hat{a}^{+}\hat{a}^{+}\hat{a}\,\hat{a} \right\rangle = \left\langle \hat{a}^{+}(\hat{a}\,\hat{a}^{+} - 1)\hat{a} \right\rangle = \left\langle \hat{a}^{+}\hat{a}\;\hat{a}^{+}\hat{a} \right\rangle - \left\langle \hat{a}^{+}\hat{a} \right\rangle$$

$$\left\langle \hat{a}^{+}\hat{a}^{+}\hat{a}\,\hat{a} \right\rangle = \left\langle \hat{n}^{2} \right\rangle - \left\langle \hat{n} \right\rangle \quad \text{where} \quad \hat{n} = \hat{a}^{+}\hat{a}$$

The correlation requires detecting two photons, so if we detect one, we have to take that into consideration in the accounting.

In terms of the variance of the photon number:

$$\sigma^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2$$

$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}$$

The classical result says:

$$= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

The quantum correlation function can be zero, as the detection changes the number of photons in the field. This is related to the variance properties: is the variance larger or smaller than the mean (Poissonian, Super-Poissonian or Sub-Poissonian).

$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}$$

At equal times the variance gives

$g^{(2)}(0)=1$ Poissonian

$g^{(2)}(0)>1$ Super-Poissonian

$g^{(2)}(0)<1$ Sub-Poissonian

The slope at time=0:

$g^{(2)}(0)>g^{(2)}(0^+)$ Bunched

$g^{(2)}(0)<g^{(2)}(0^+)$ Antibunched

Classically we can not have Sub-Poissonian nor Antibunched.

Quantum Correlations (Glauber):

$$g^{(2)}(\tau) = \frac{\left\langle :\hat{I}(t)\,\hat{I}(t+\tau): \right\rangle}{\left\langle \hat{I}(t) \right\rangle^{2}}$$

$$g^{(2)}(\tau) = \frac{\left\langle :\hat{I}(\tau): \right\rangle_{c}}{\left\langle :\hat{I}: \right\rangle}$$

If we detect a photon at time t , $g^{(2)}(\tau)$ gives the probability of detecting a second photon after a time τ .

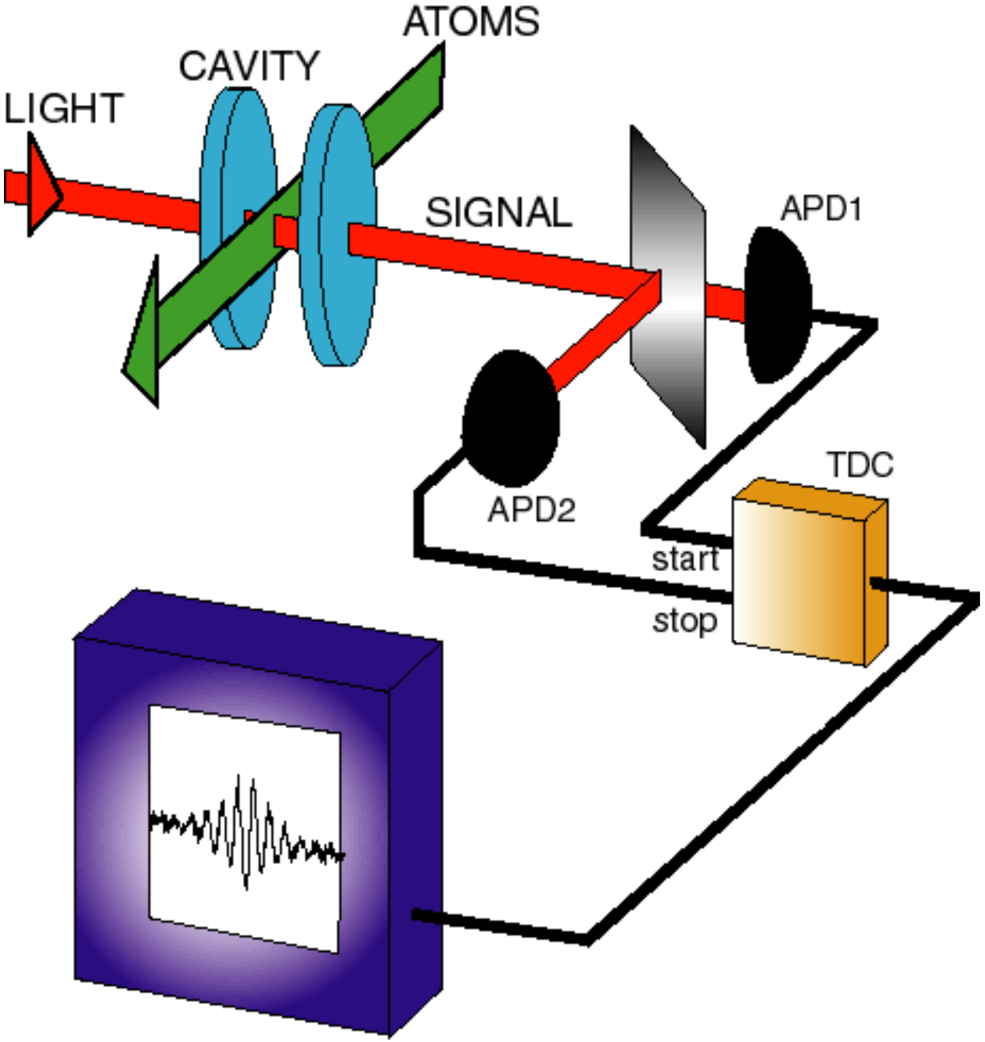
Correlation functions as conditional measurements in quantum optics.

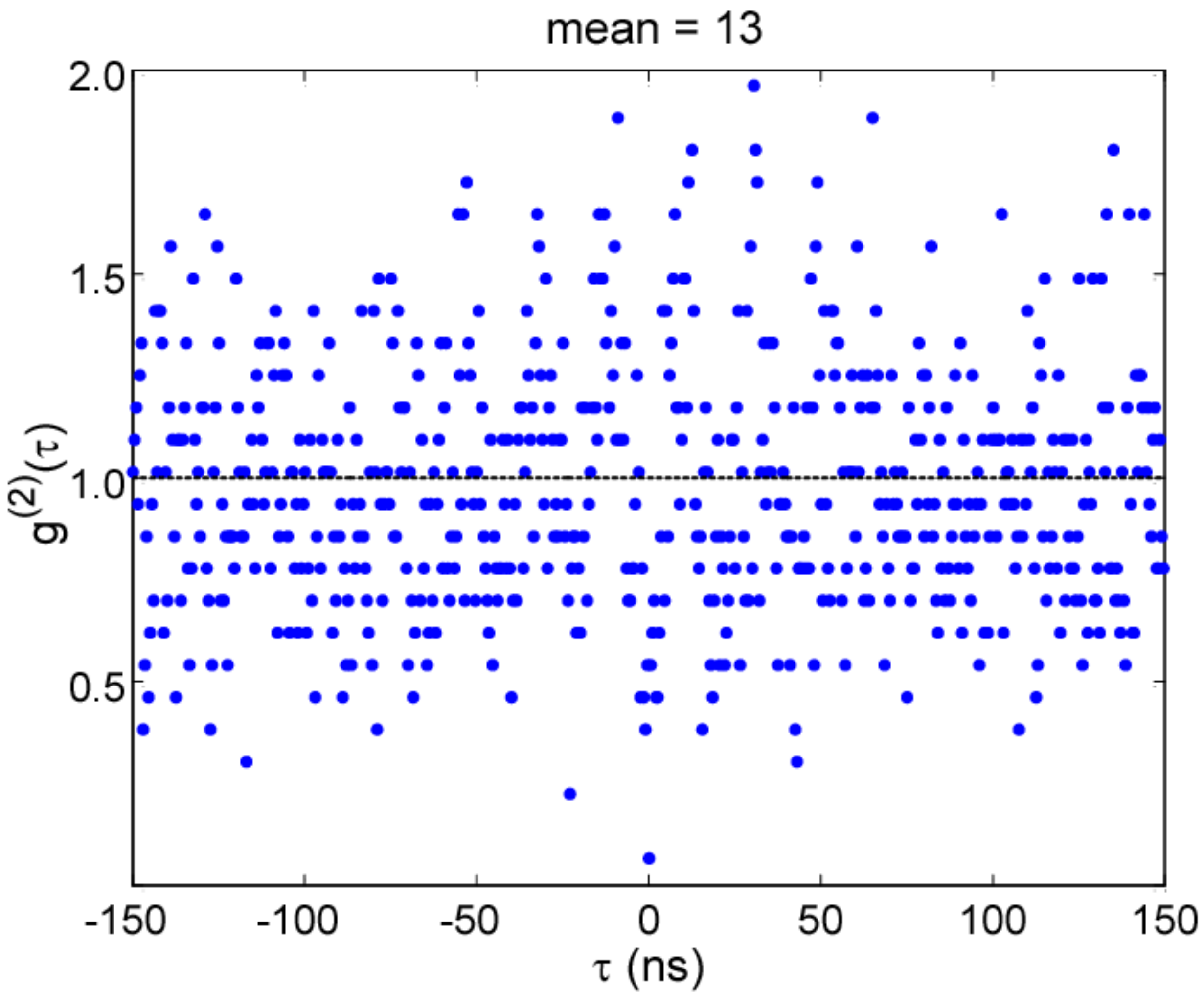
The detection of the first photon gives the initial state that is going to evolve in time.

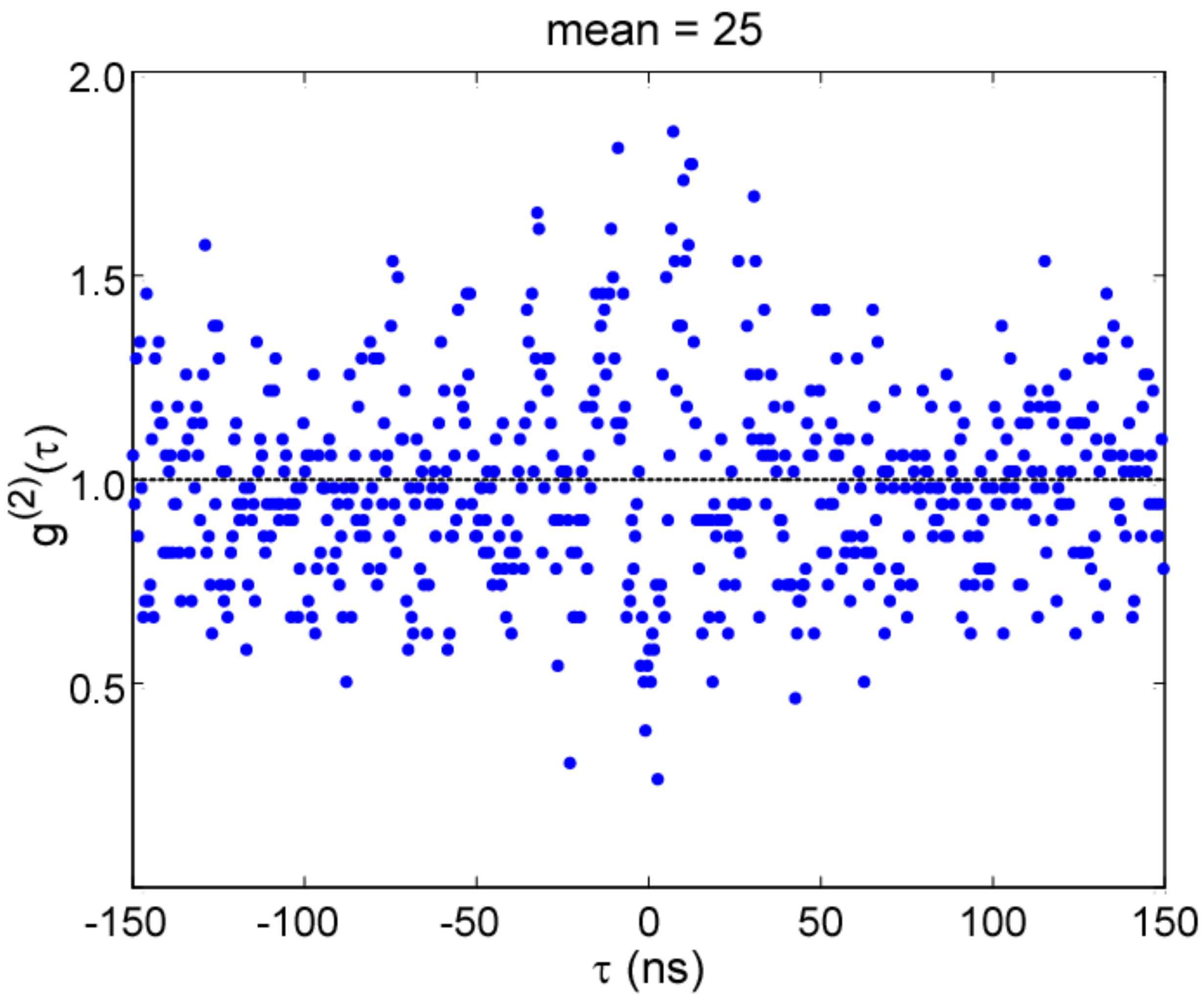
This may sound as Bayesian probabilities.

$g^{(1)}(\tau)$ Interferograms.

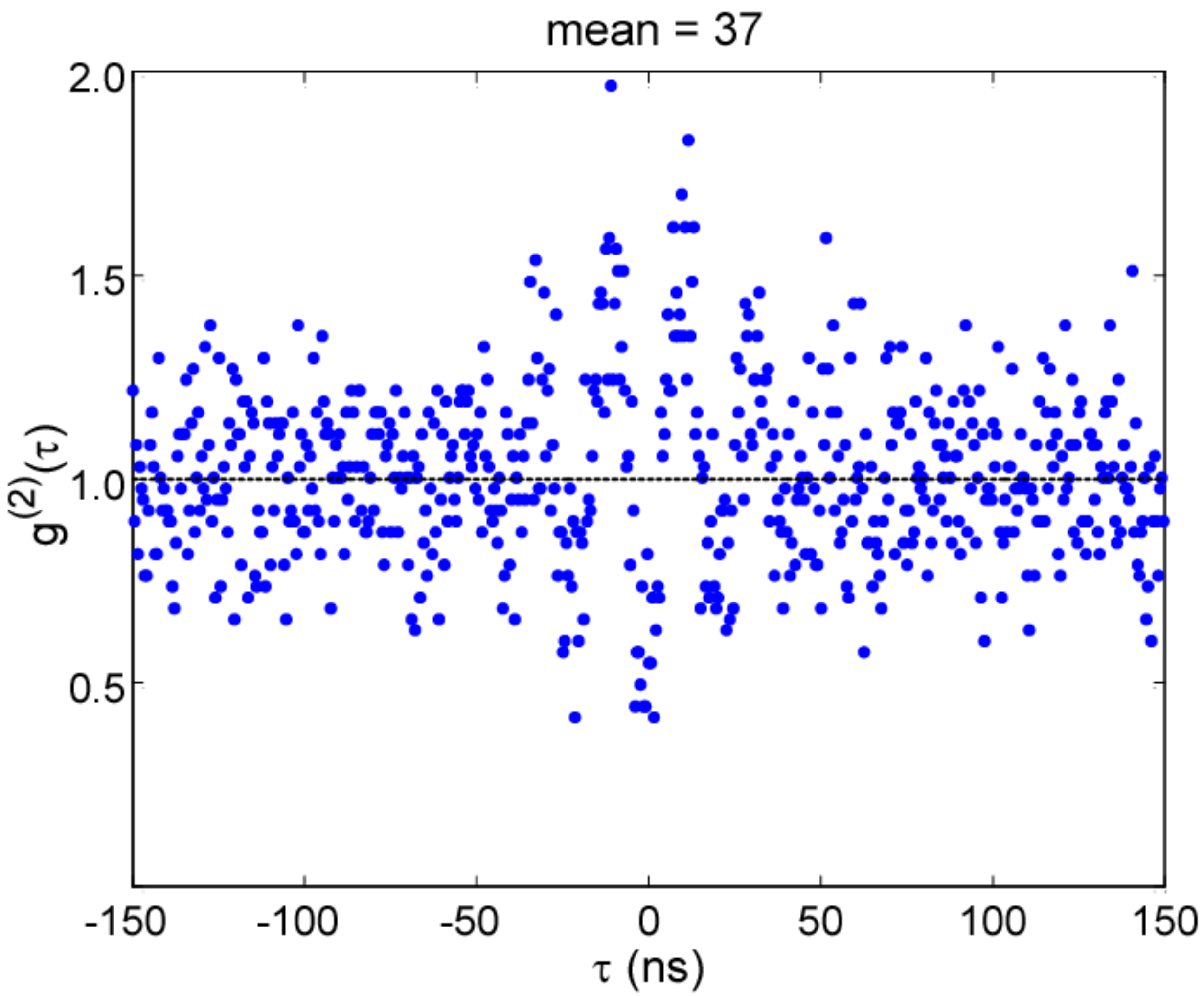
$g^{(2)}(\tau)$ Hanbury-Brown and Twiss.


ATOMS
CAVITY
LIGHT
SIGNAL
APD1
APD2
TDC
start
stop


mean = 13
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



mean = 25
2.0
1.5
1.0
0.5
g(2)(τ)
-150
-100
-50
0
50
100
150
τ (ns)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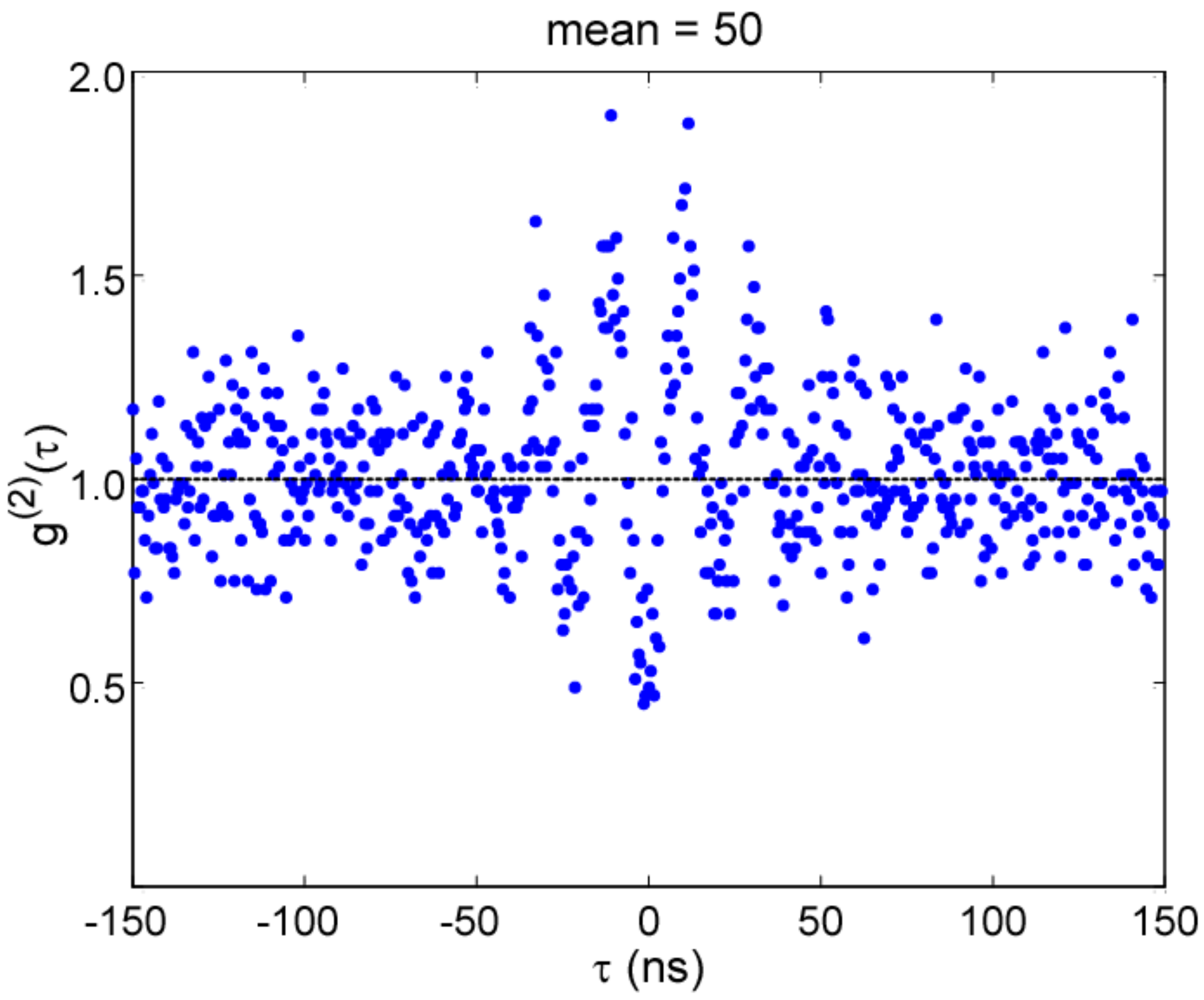
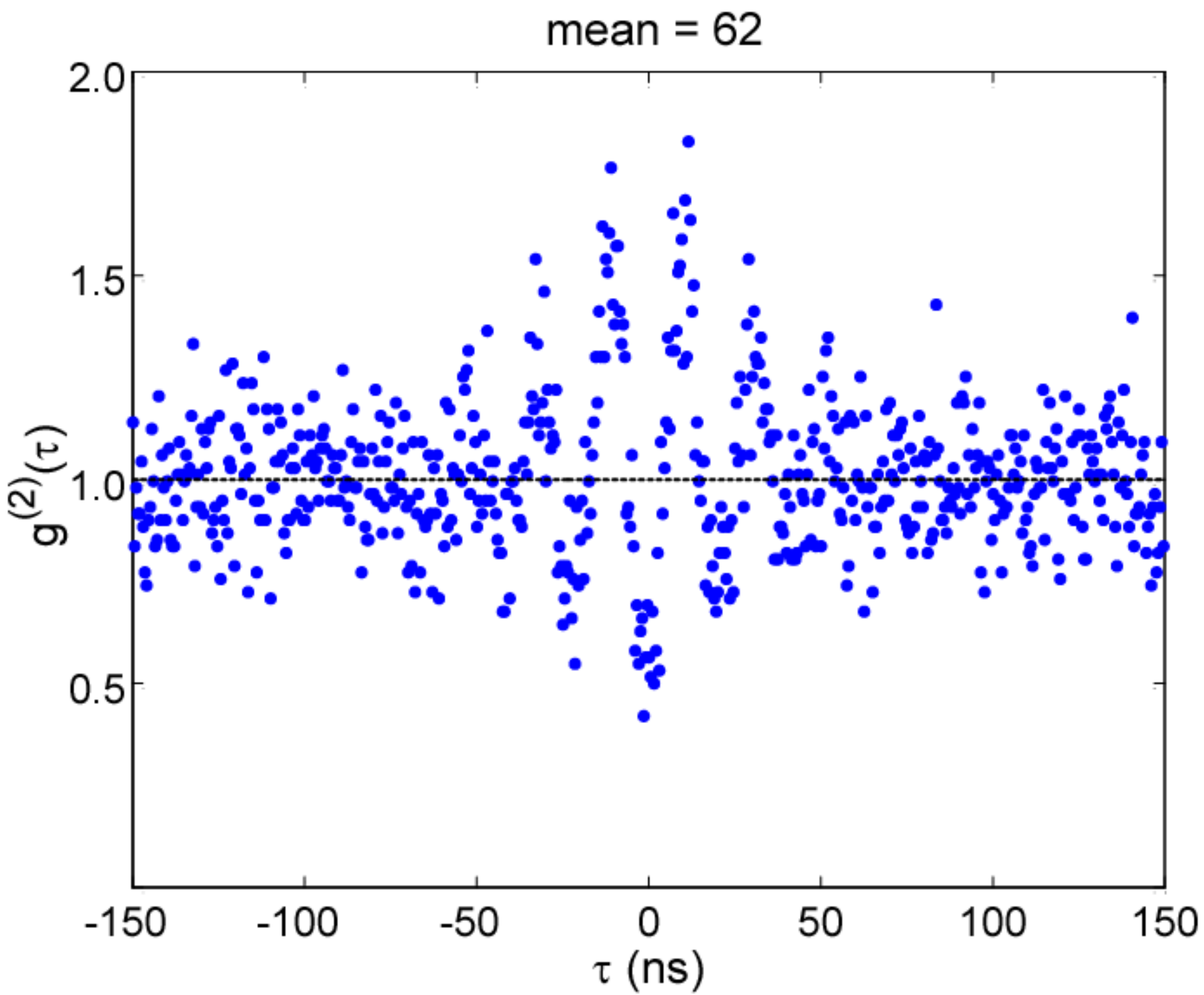
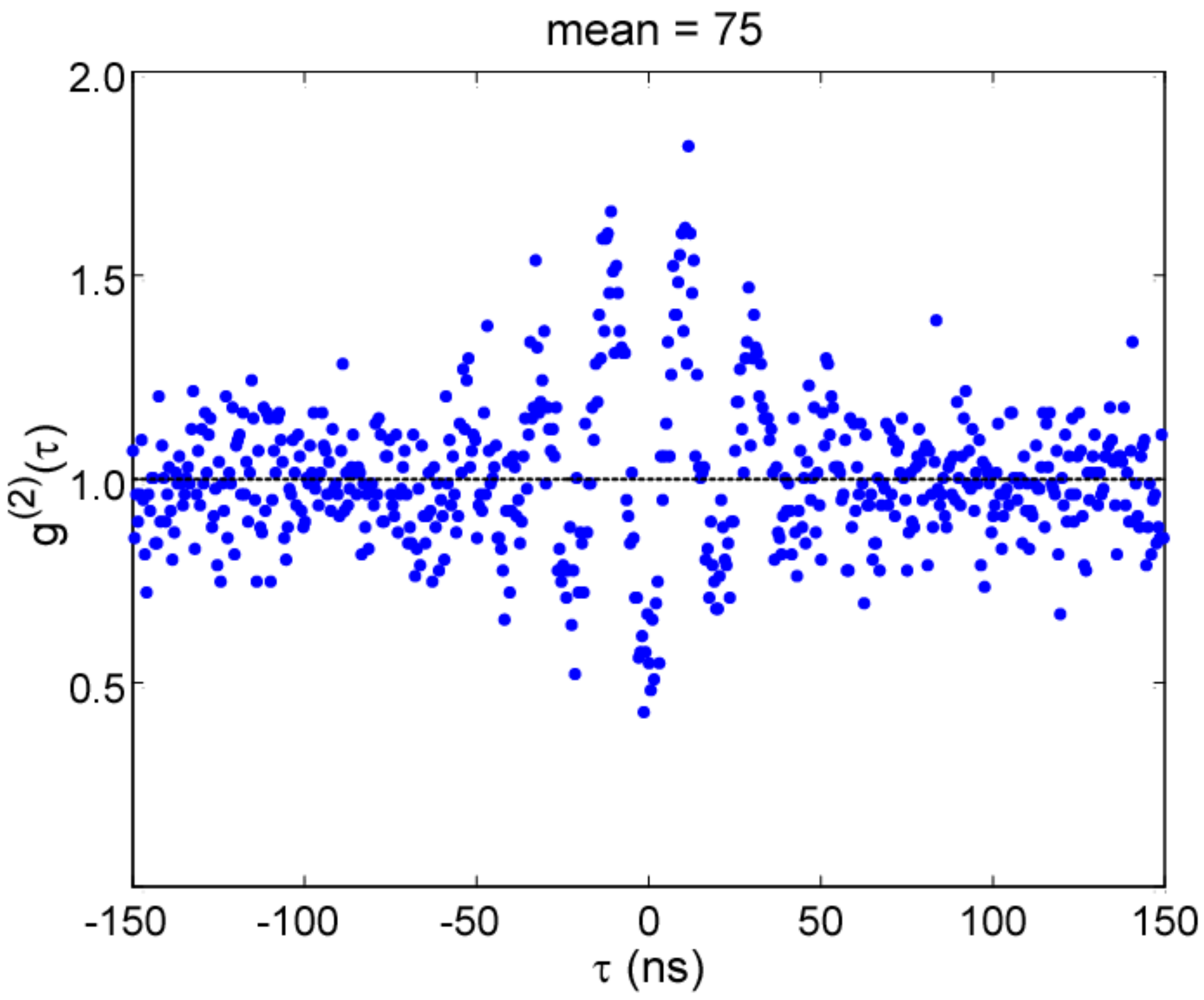
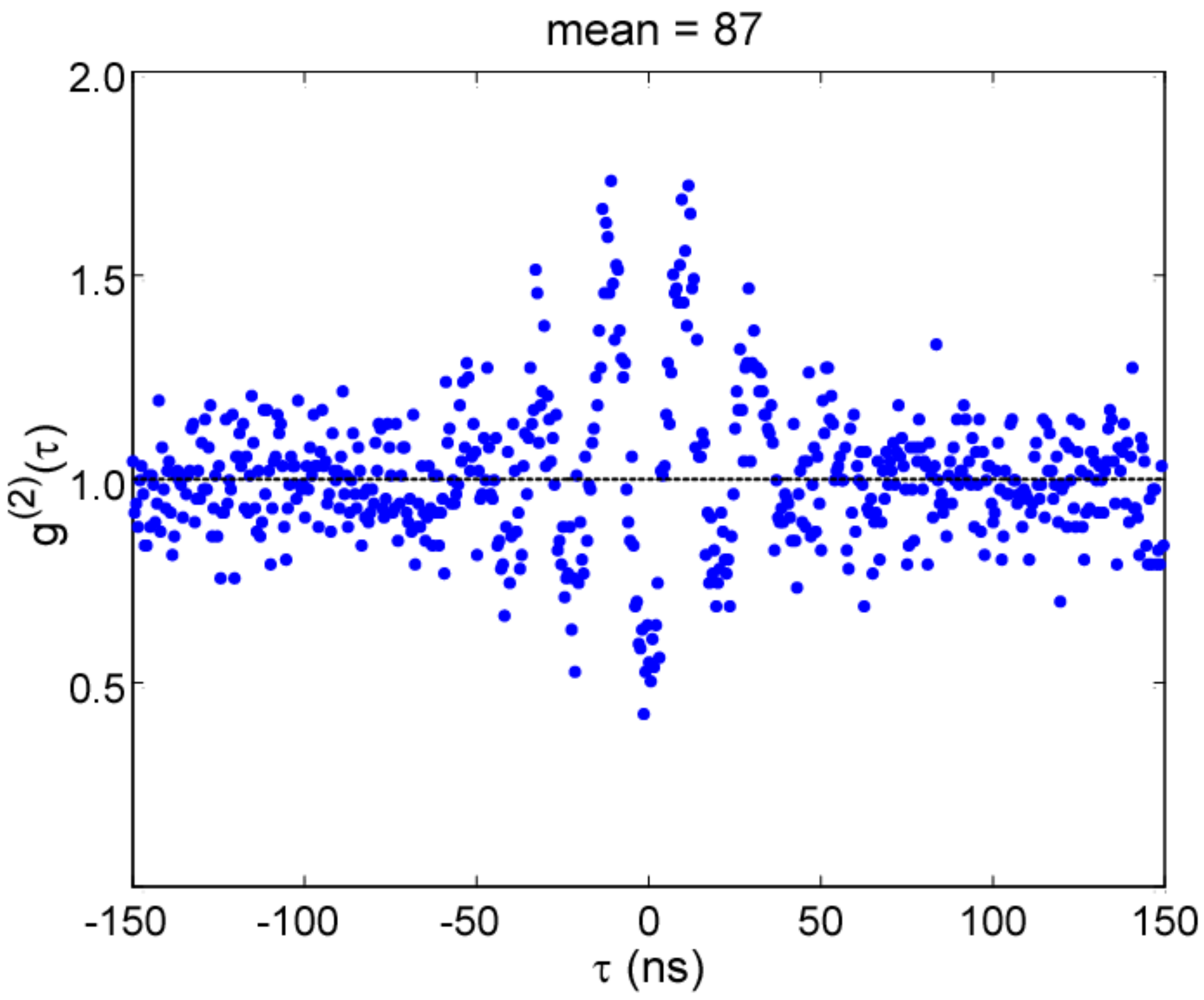
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

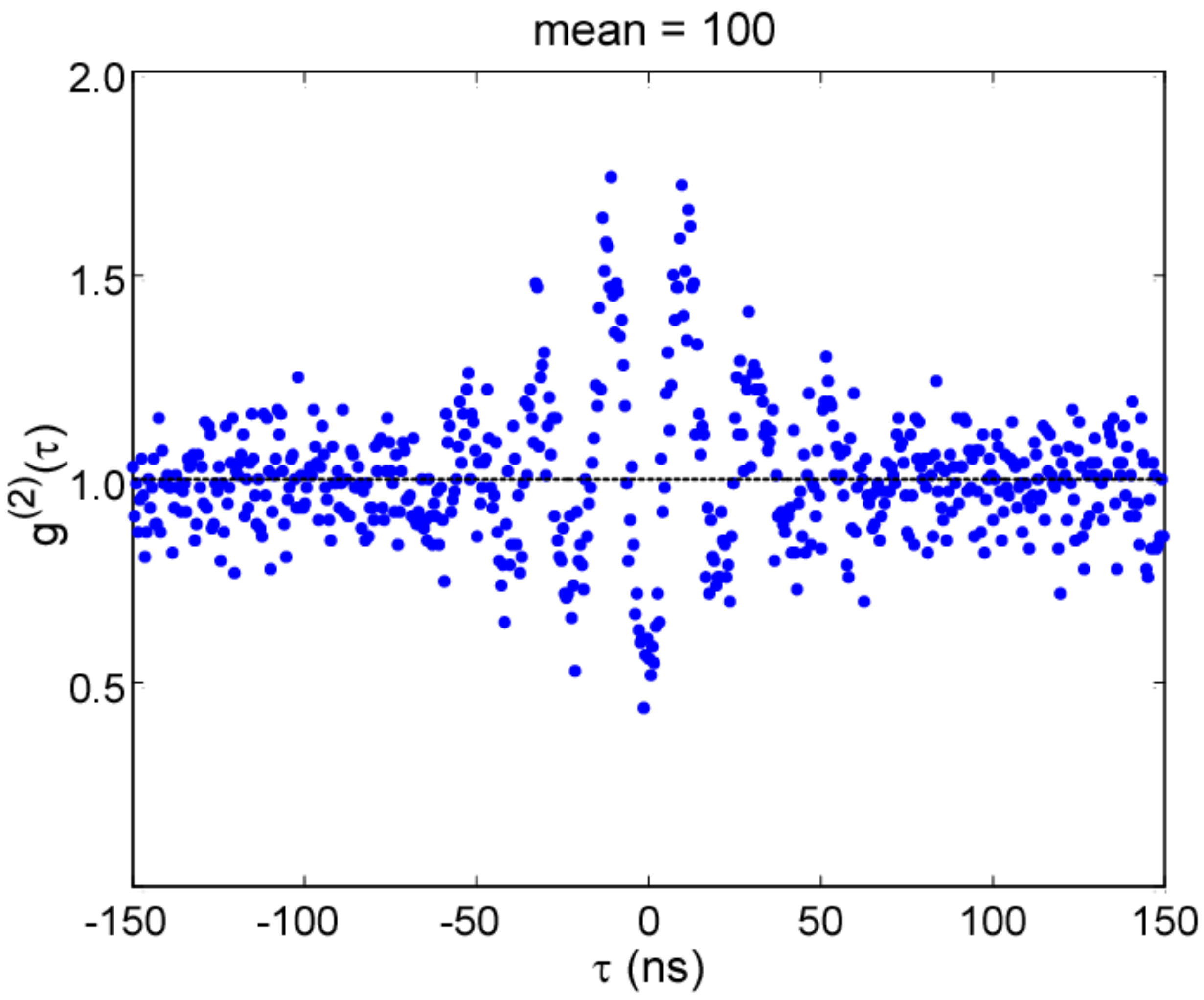


mean = 50
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

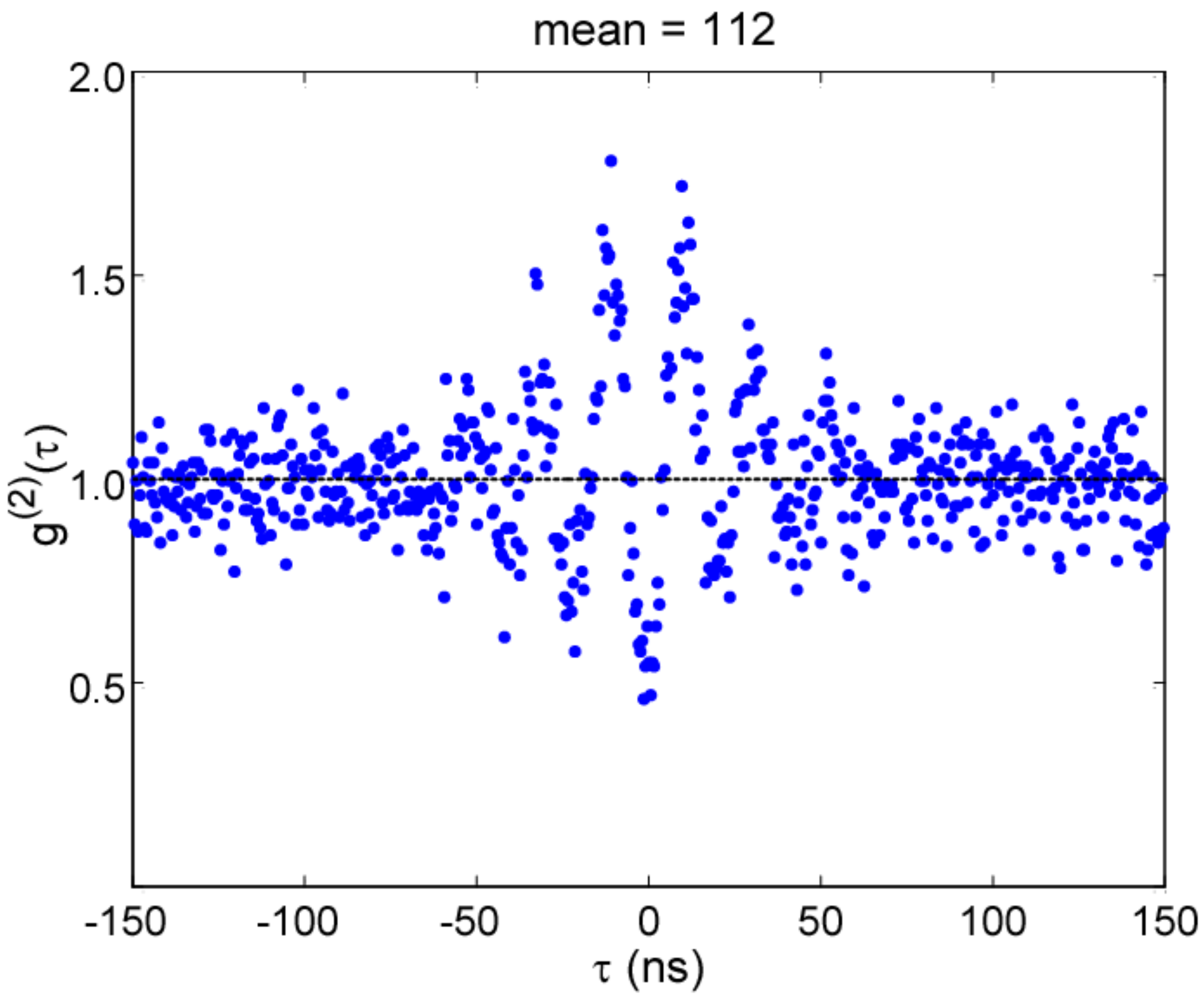

mean = 62
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


mean = 75
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


mean = 87
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



mean = 100
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


mean = 112
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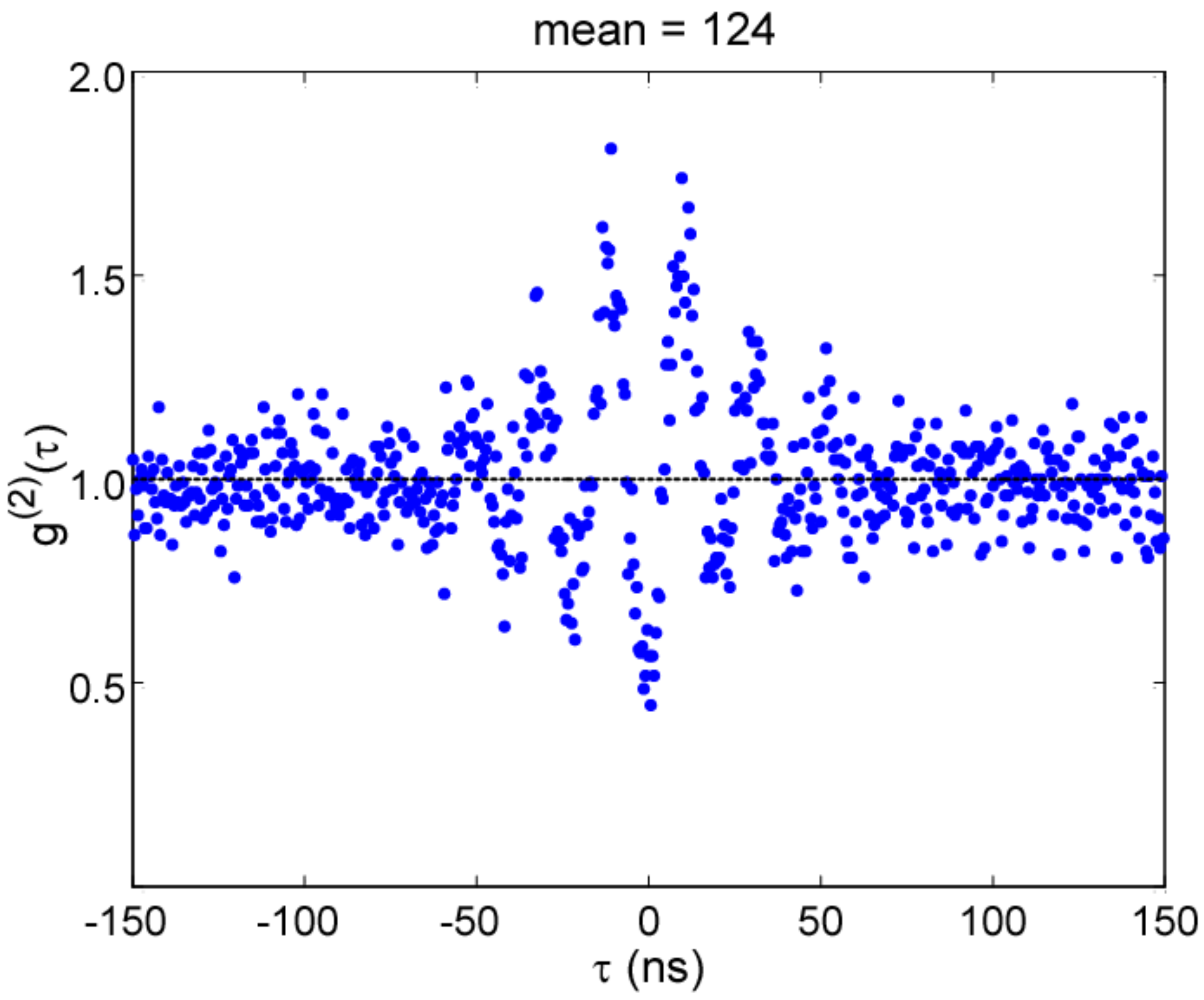
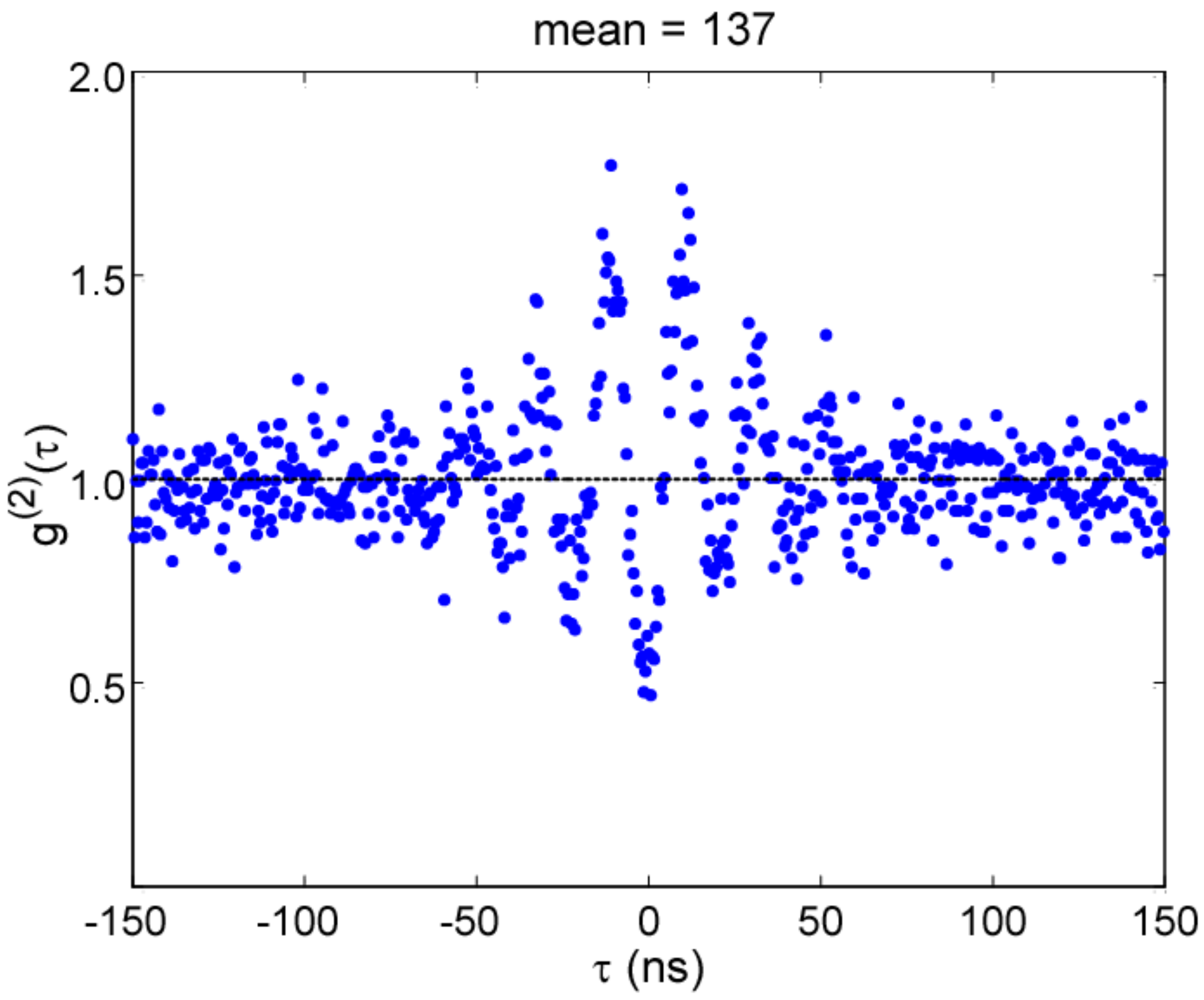
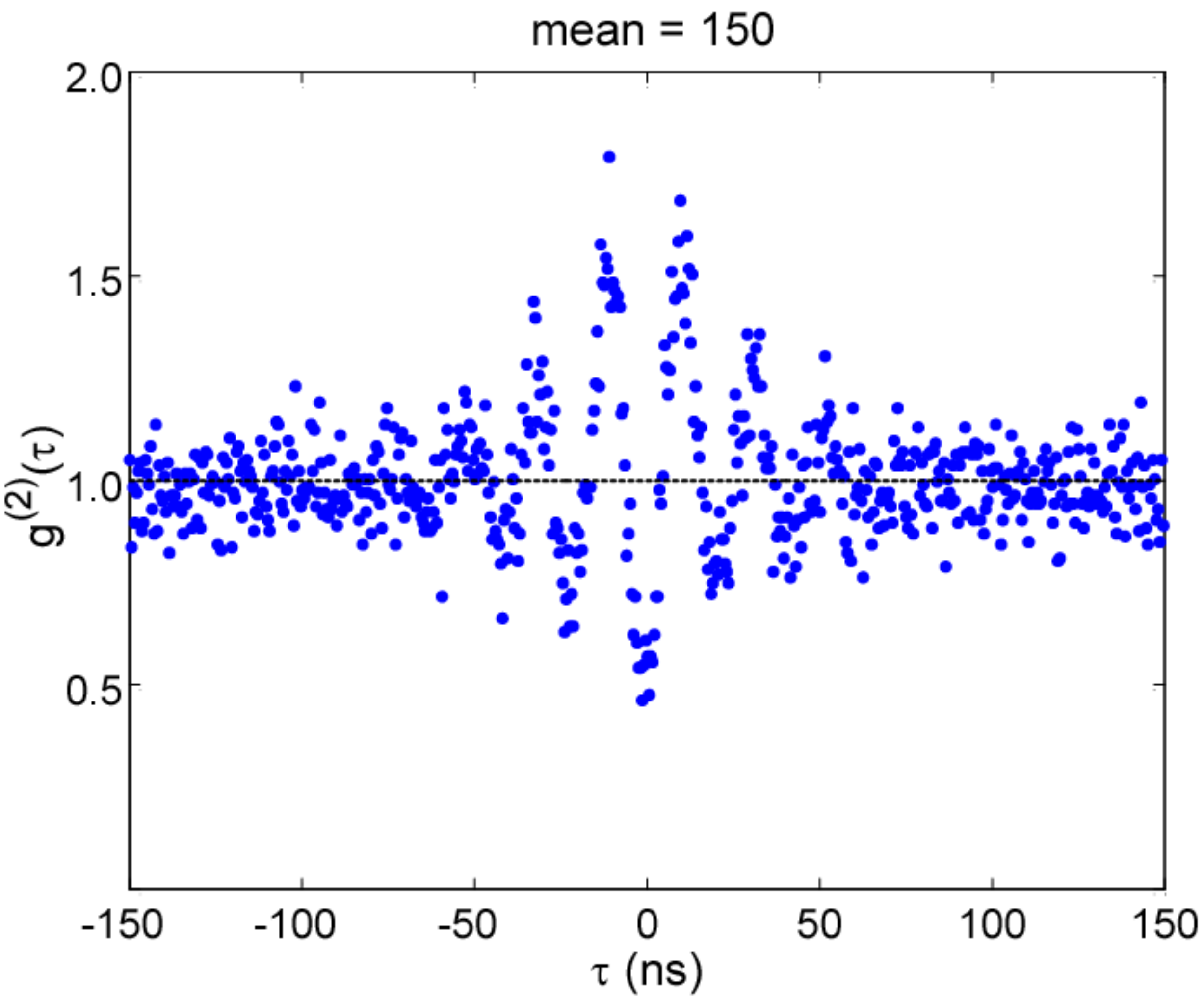
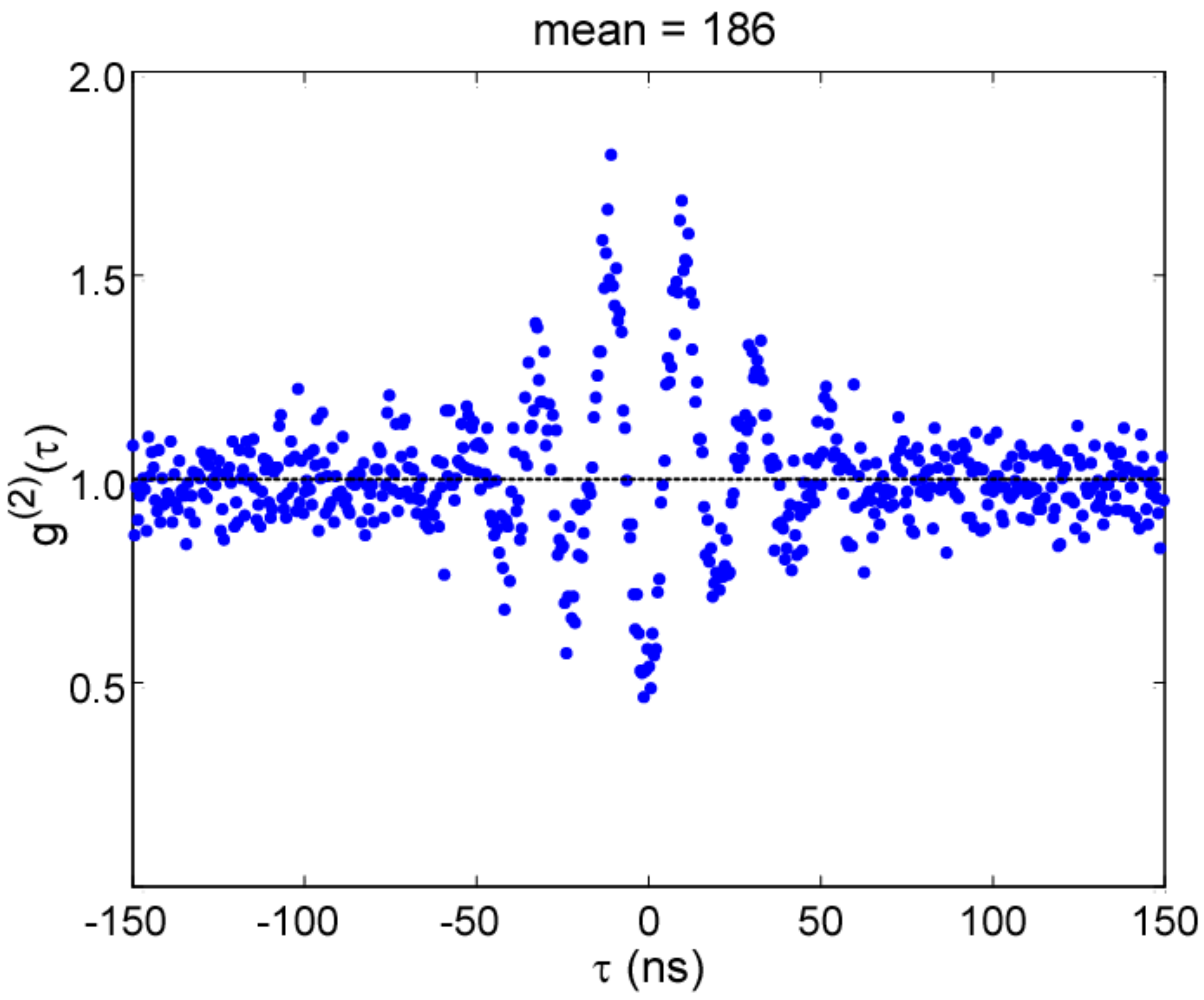
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


mean = 124
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

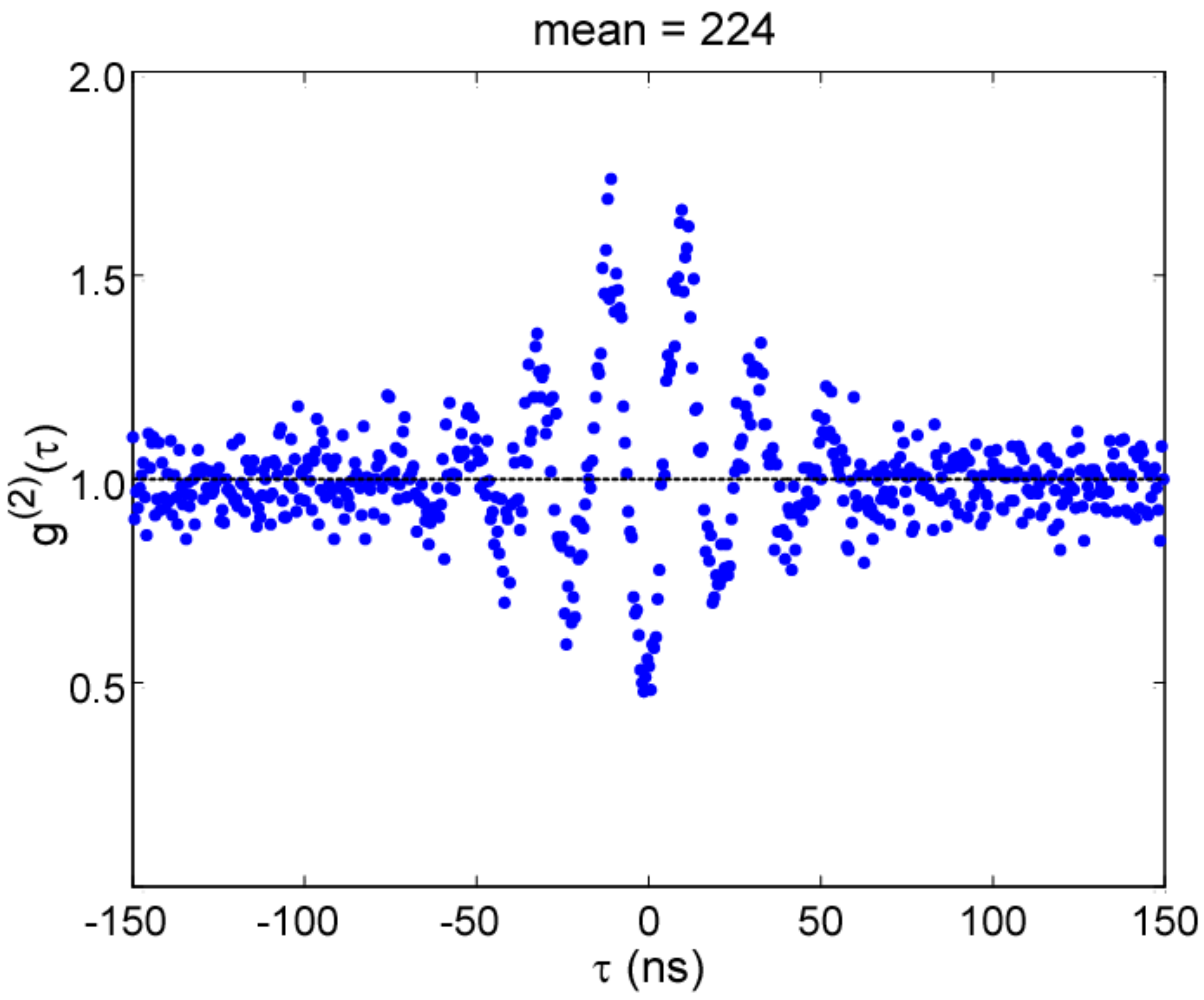

mean = 137
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



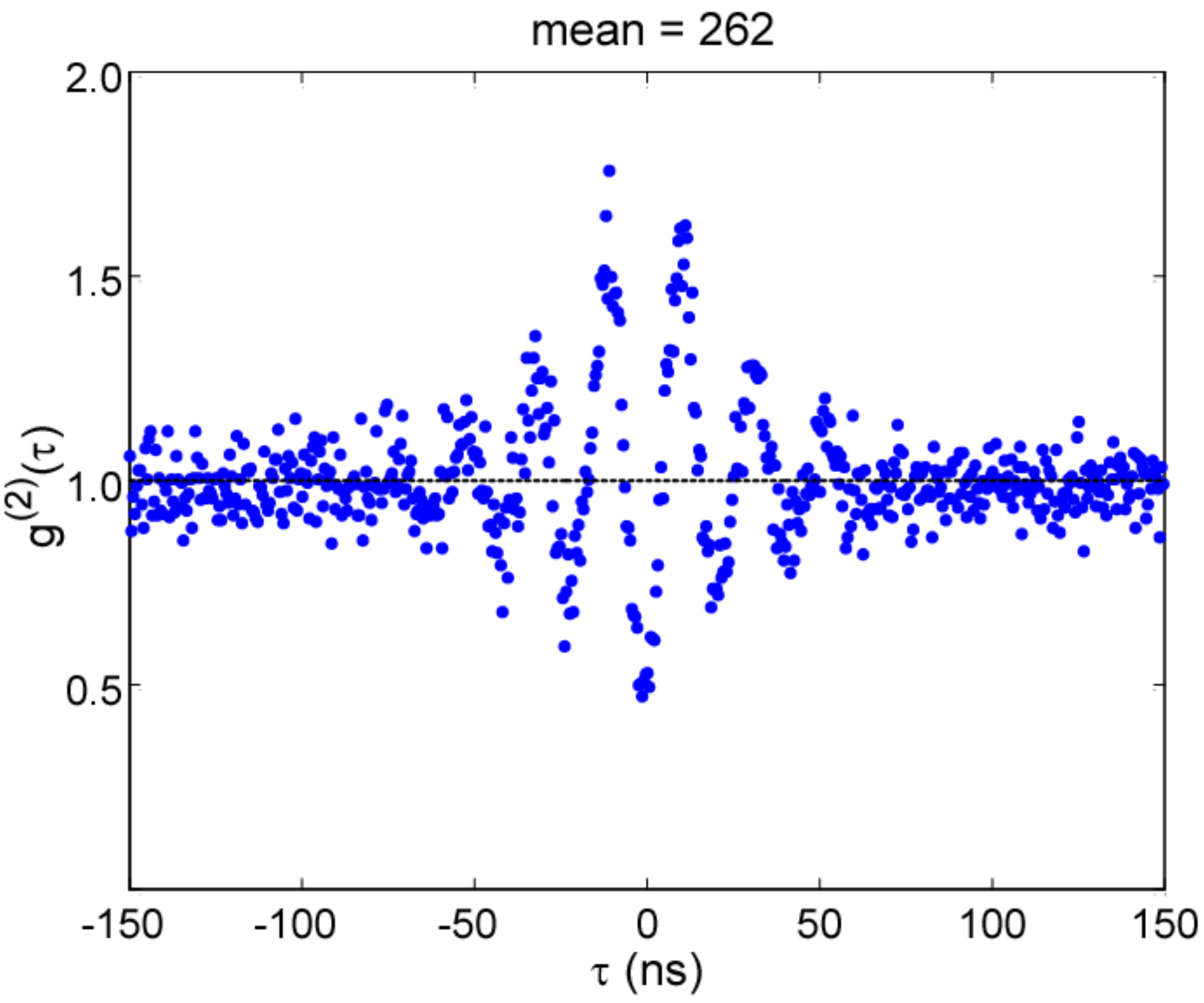
mean = 150
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



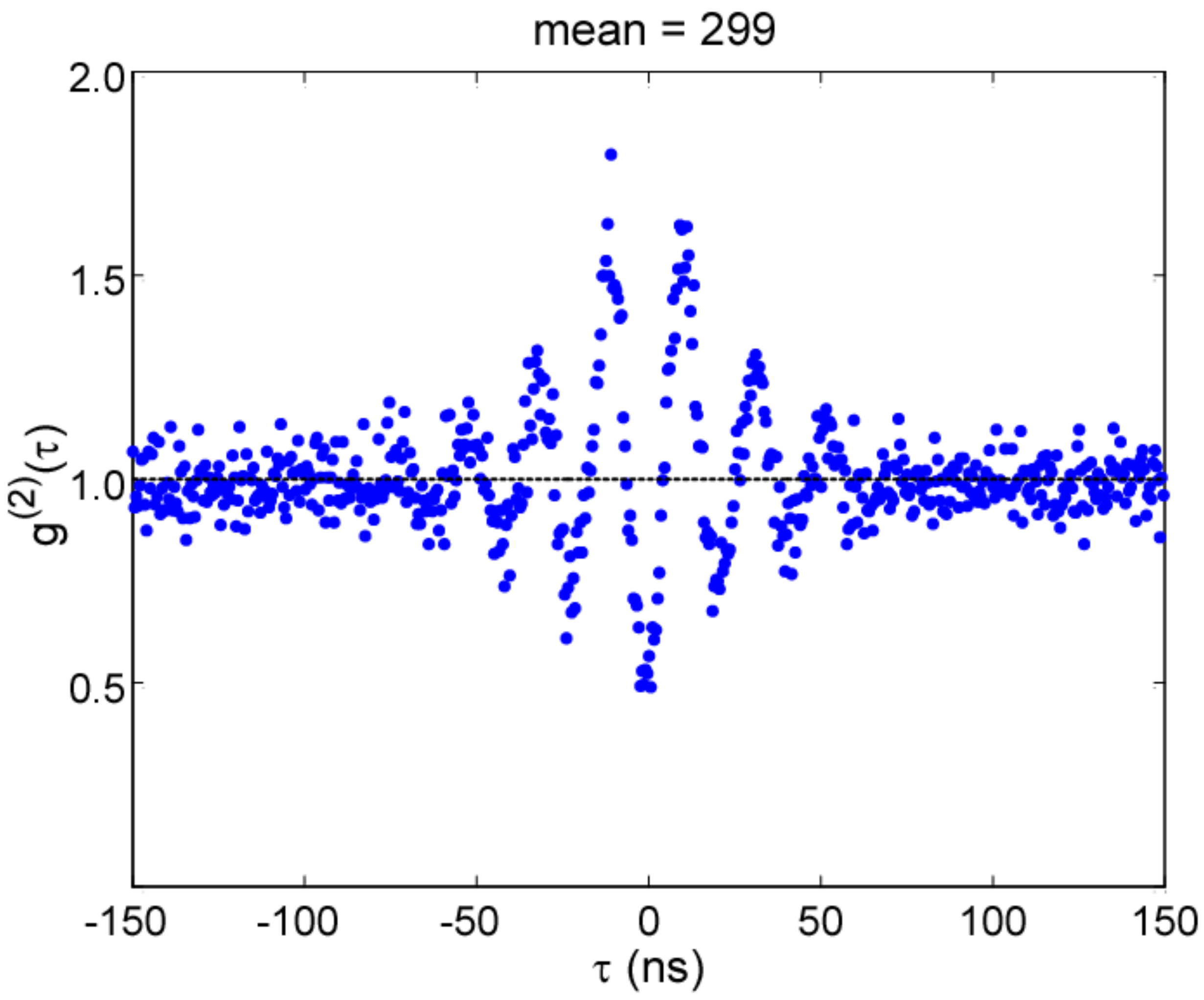
mean = 186
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

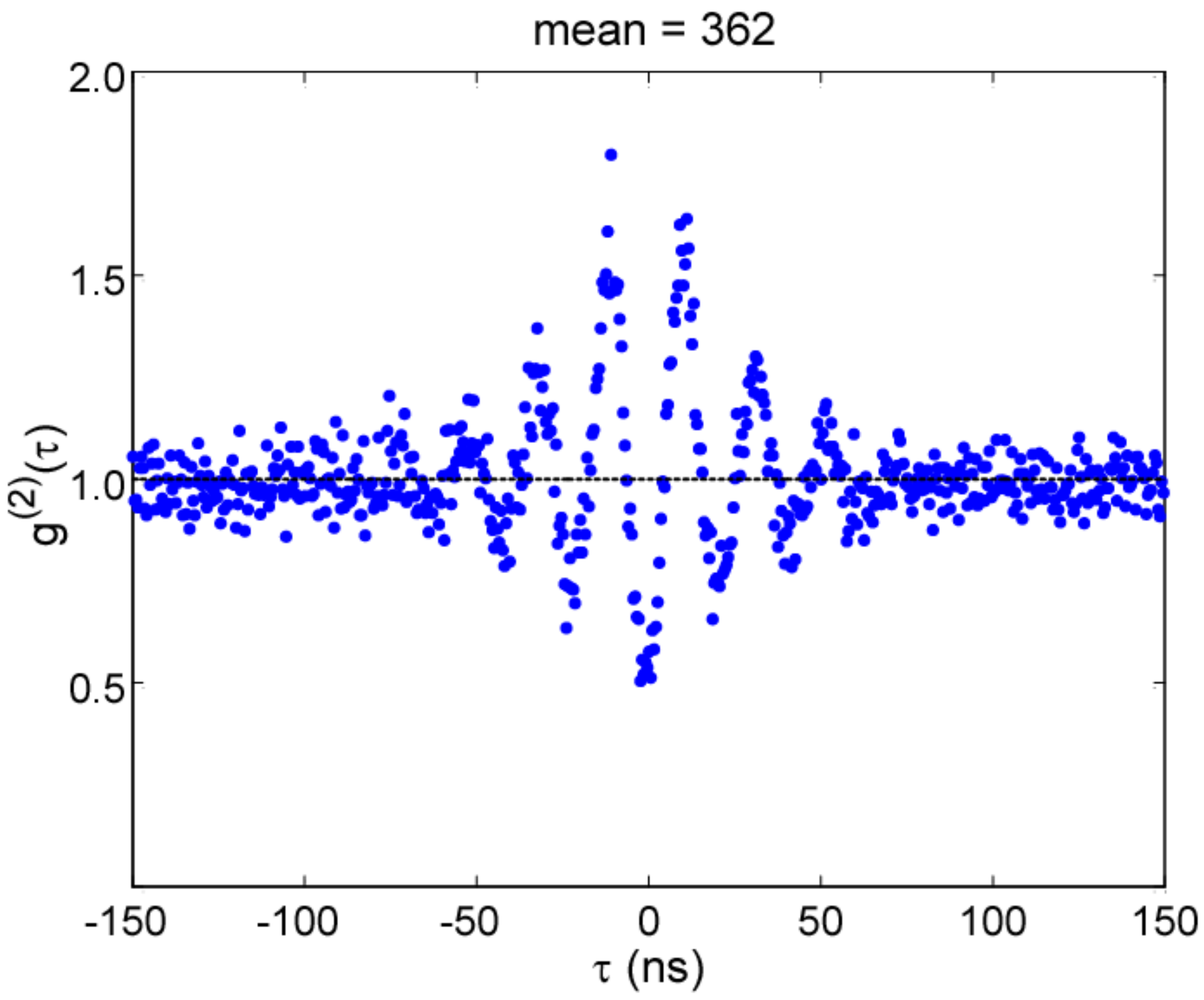


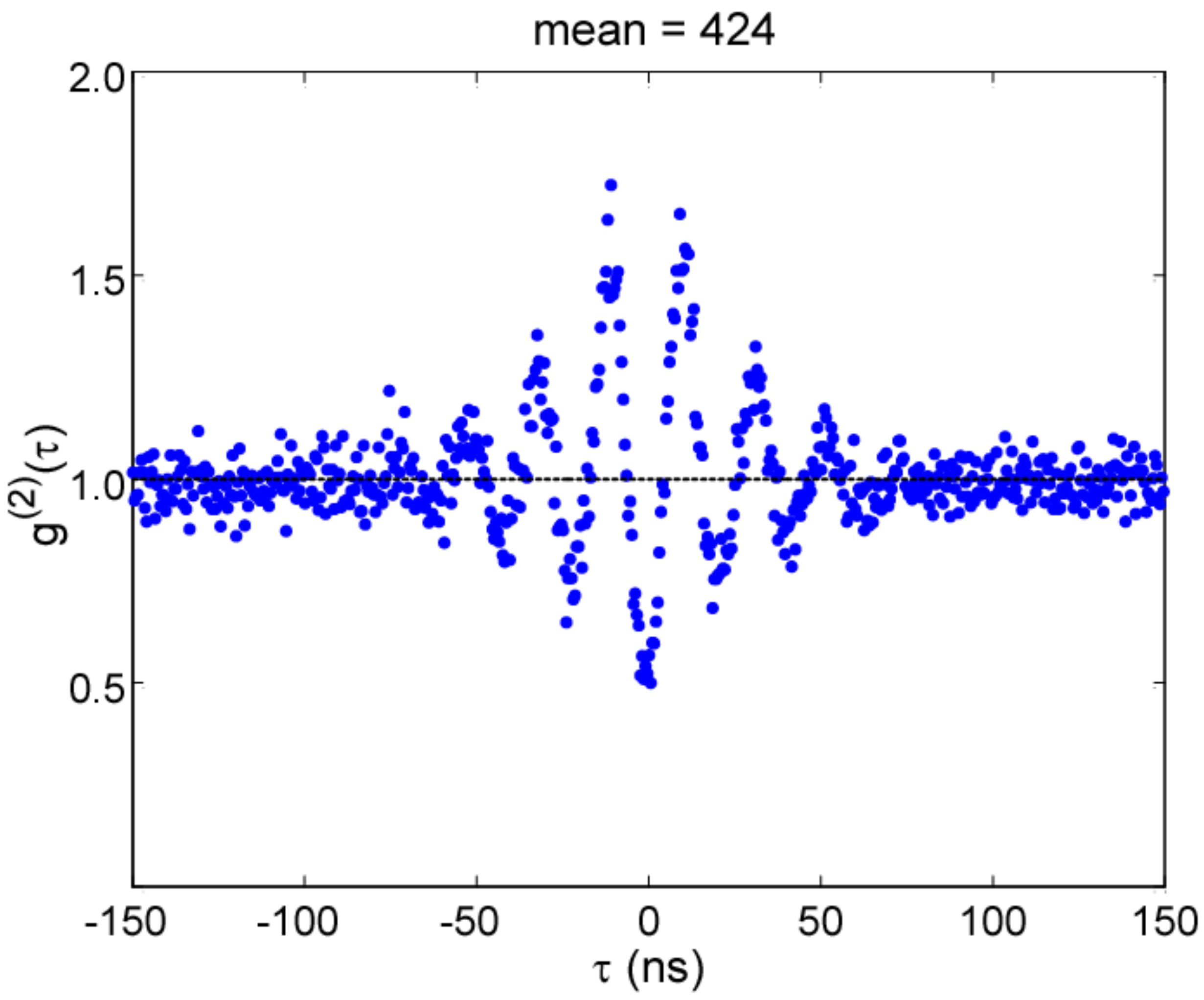
mean = 224
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



mean = 262
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


mean = 299
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


mean = 362
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



mean = 424
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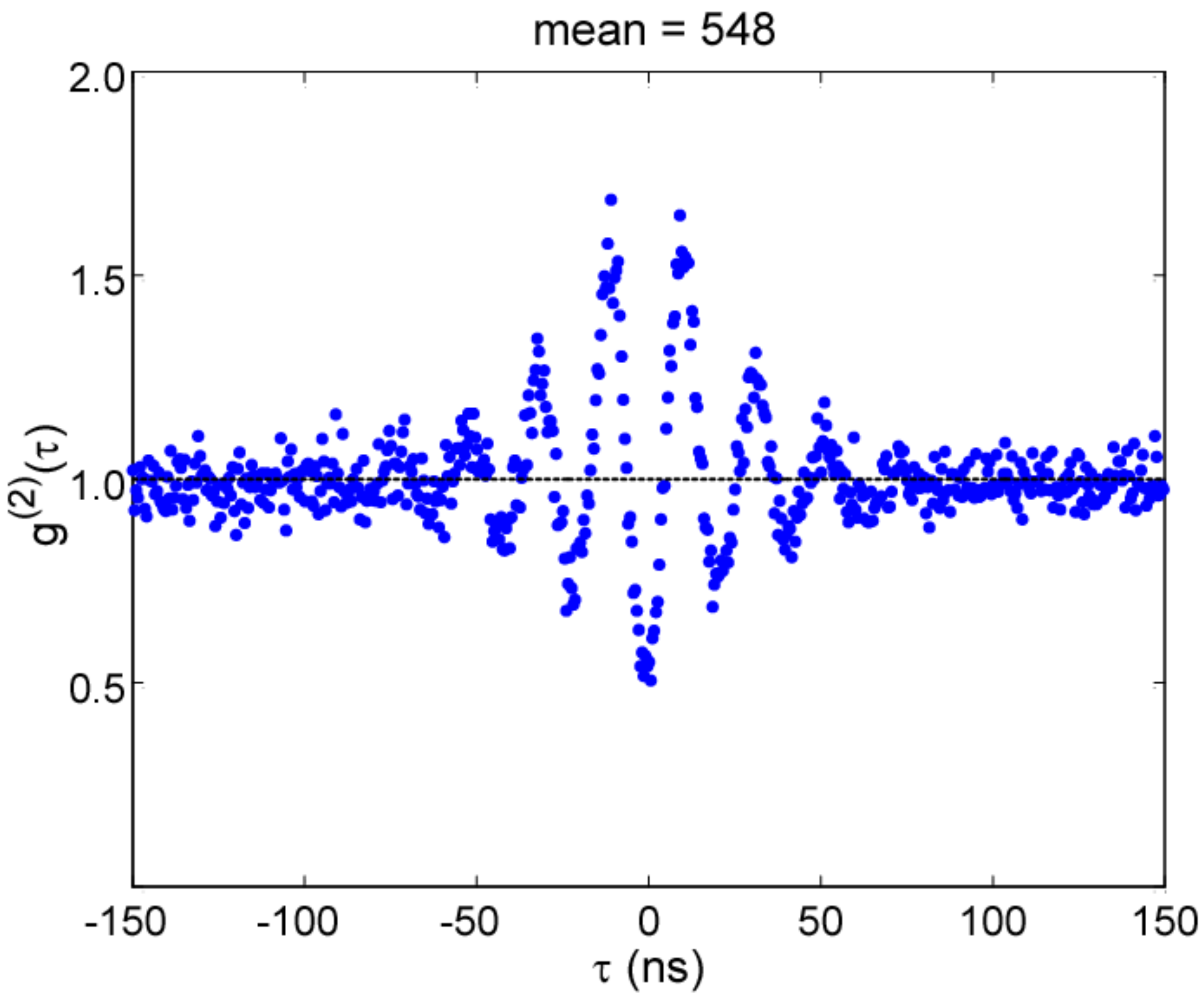
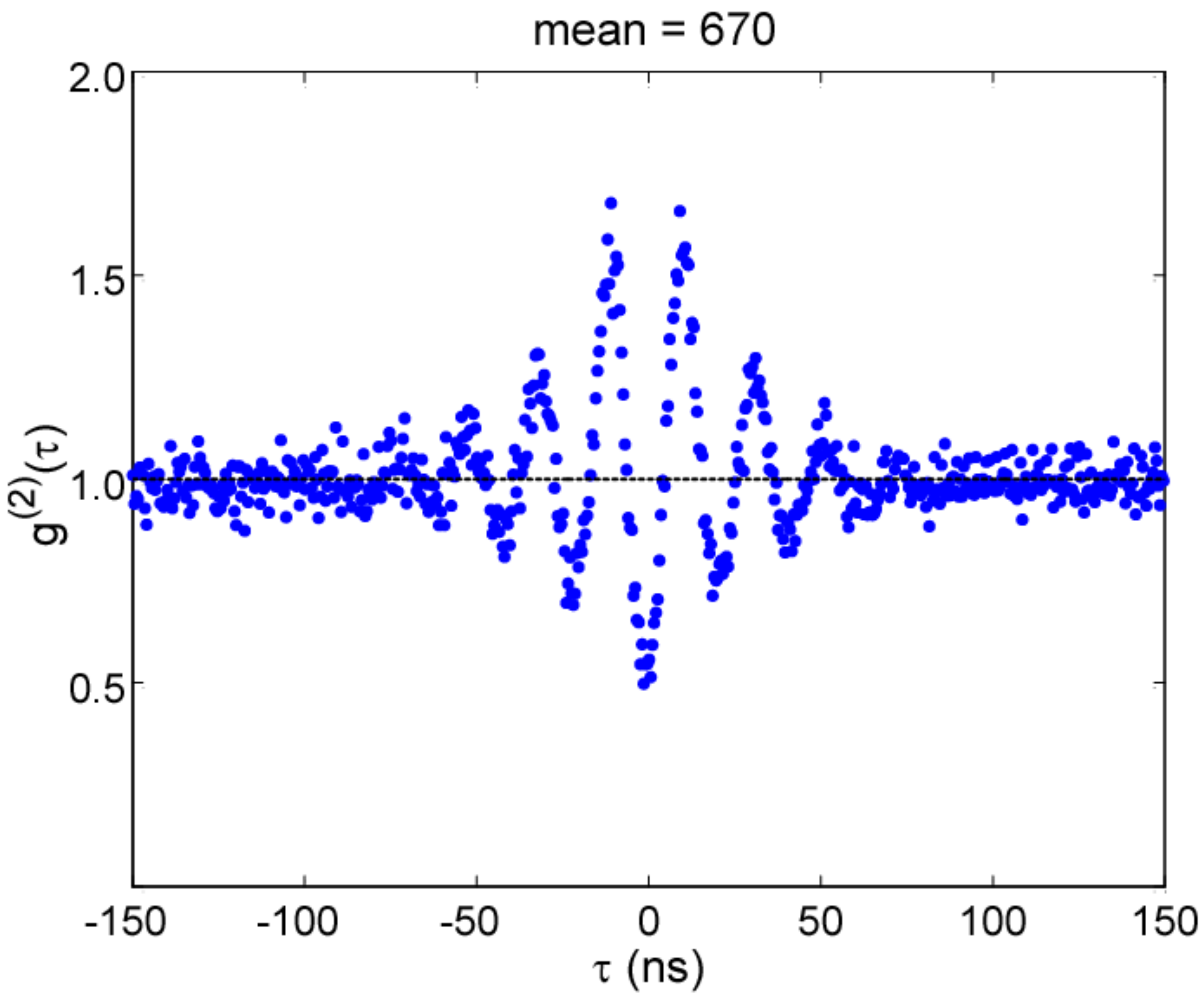
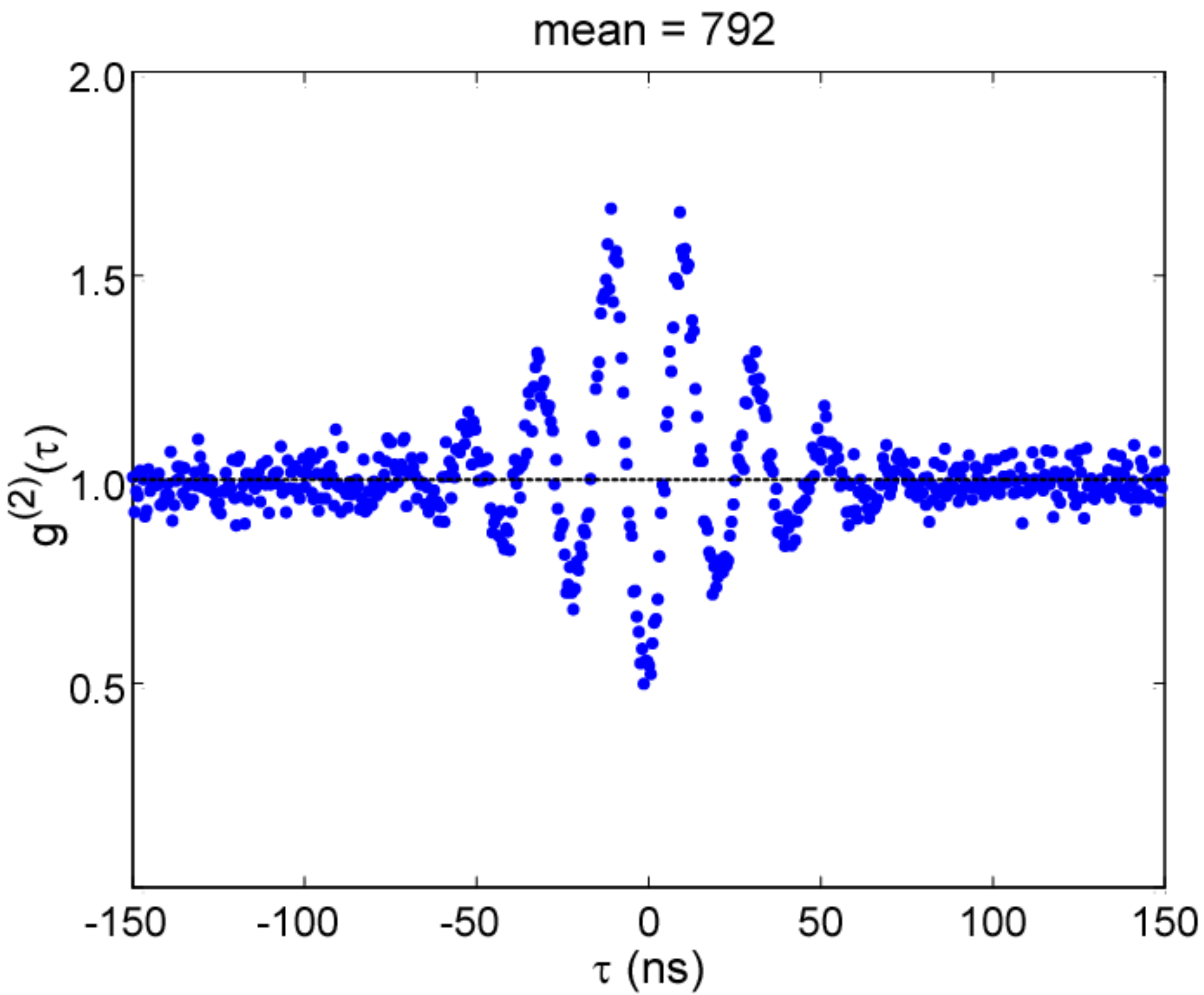
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



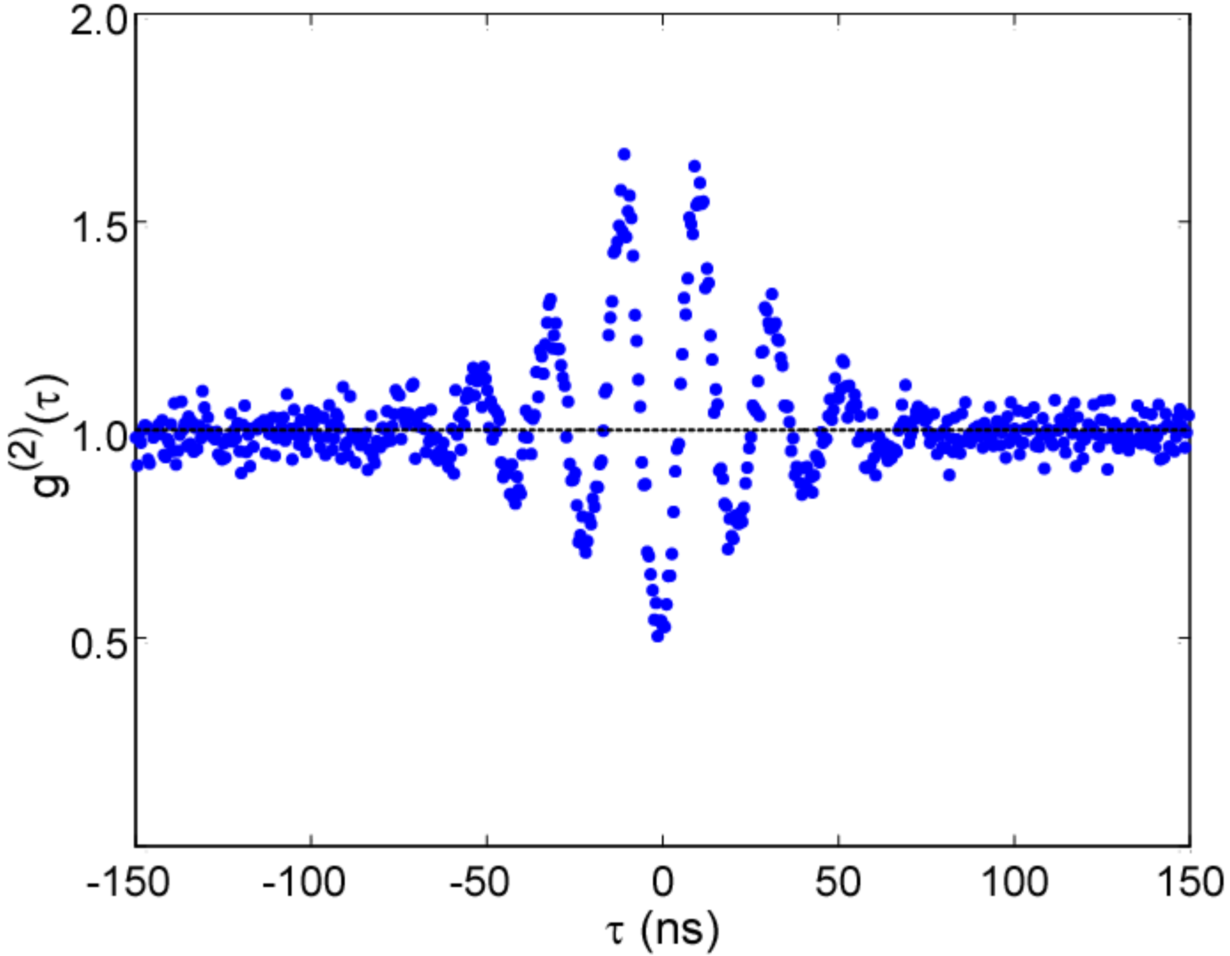
mean = 548
2.0
1.5
1.0
0.5
g(2)(τ)
-150
-100
-50
0
50
100
150
τ (ns)

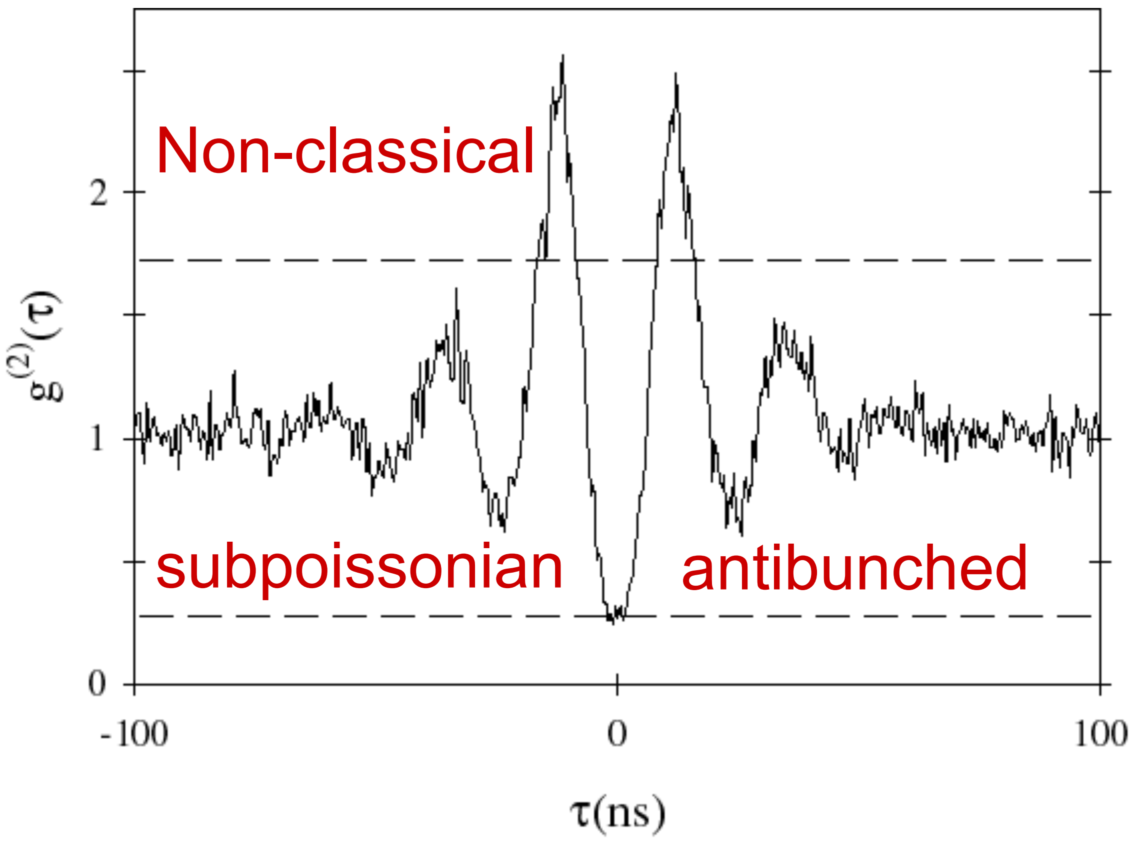


mean = 670
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


mean = 792
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

7 663 536 starts mean = 913 1 838 544 stops





Classically $g^{(2)}(0) > g^{(2)}(\tau)$, $g^{(2)}(0) > g^{(2)}(0+\varepsilon)$ negative slope antibunching, $g^{(2)}(0)<1$ subpoissonian, also $|g^{(2)}(0)-1| > |g^{(2)}(\tau)-1|$

Conditional evolution of the state one atom

$$|\Psi_{ss}\rangle = |0,g\rangle + \lambda|1,g\rangle - \frac{2g}{\gamma}\lambda|0,e\rangle + \frac{\lambda^2 pq}{\sqrt{2}}|2,g\rangle - \frac{2g\lambda^2 q}{\gamma}|1,e\rangle$$

$$\lambda = \langle \hat{a} \rangle, \quad p = p(g,\kappa,\gamma) \text{ and } q = q(g,\kappa,\gamma) \qquad \lambda = \frac{\varepsilon}{\kappa}\left(\frac{1}{1+2C}\right)$$

$$p = 1 - 2C_1', \quad q = (1+2C)/(1+2C-2C_1') \quad \text{with } C_1' = C_1(1+\gamma/2\kappa)^{-1}$$

$$\hat{a}|\Psi_{ss}\rangle \Rightarrow |\Psi_{conditioned}\rangle = |0,g\rangle + \lambda pq|1,g\rangle - \frac{2g\lambda q}{\gamma}|0,e\rangle$$

$$|\langle 0,g|\hat{a}^2|\psi\rangle| = |\lambda^2 pq|^2$$

Field atomic polarization

$$g^{(2)}(0) = |pq|^2 = 1 - \frac{4C_1^2}{(1+\gamma/2\kappa)(1-2C_1)-2C_1}$$

For N non-interacting atoms

$$|\chi(t)\rangle = |00\rangle + A_1(t)|10\rangle + A_2(t)|01\rangle + A_3(t)|20\rangle + A_4(t)|11\rangle + A_5(t)|02\rangle$$

Equations of motion of the coefficients

$$\dot{A}_1 = -\kappa A_1 + \sqrt{N} g A_2 + \mathscr{E}$$

$$\dot{A}_2 = -(\gamma/2) A_2 - \sqrt{N} g A_1$$

$$\dot{A}_3 = -2\kappa A_3 + \sqrt{2}\sqrt{N} g A_4 + \sqrt{2}\mathscr{E} A_1 ,$$

$$\dot{A}_4 = -(\kappa + \gamma/2) A_4 - \sqrt{2}\sqrt{N} g A_3 + \sqrt{2}\sqrt{N-1} g A_5 + \mathscr{E} A_2 ,$$

$$\dot{A}_5 = -\gamma A_5 - \sqrt{2}\sqrt{N-1} g A_4 .$$

Field with drive $\mathcal{E}$

Polarization

A. Low-intensity theory for $g^{(2)}(\tau)$

$$g^{(2)}(\tau)=|1+\mathcal{A}\mathcal{F}(\tau)|^2, \tag{17}$$

where $\mathcal{F}$ is a decaying oscillation,

$$\mathcal{F}=e^{-\beta\tau}[\cos(\Omega_0\tau)+(\beta/\Omega_0)\sin(\Omega_0\tau)], \tag{18}$$

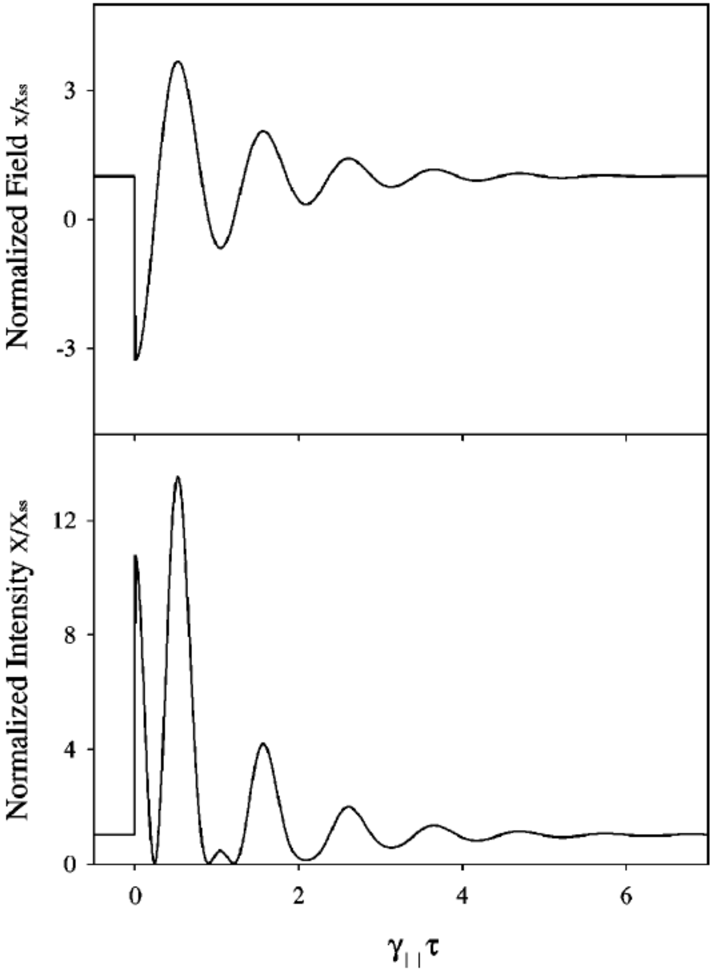
with $\beta\equiv(\kappa+\gamma_\perp)/2$ representing the average decay rate and Ω_0 the vacuum Rabi frequency in the low intensity limit:

$$\Omega_0=\sqrt{g^2N-\frac{(\kappa-\gamma_\perp)^2}{4}}.$$

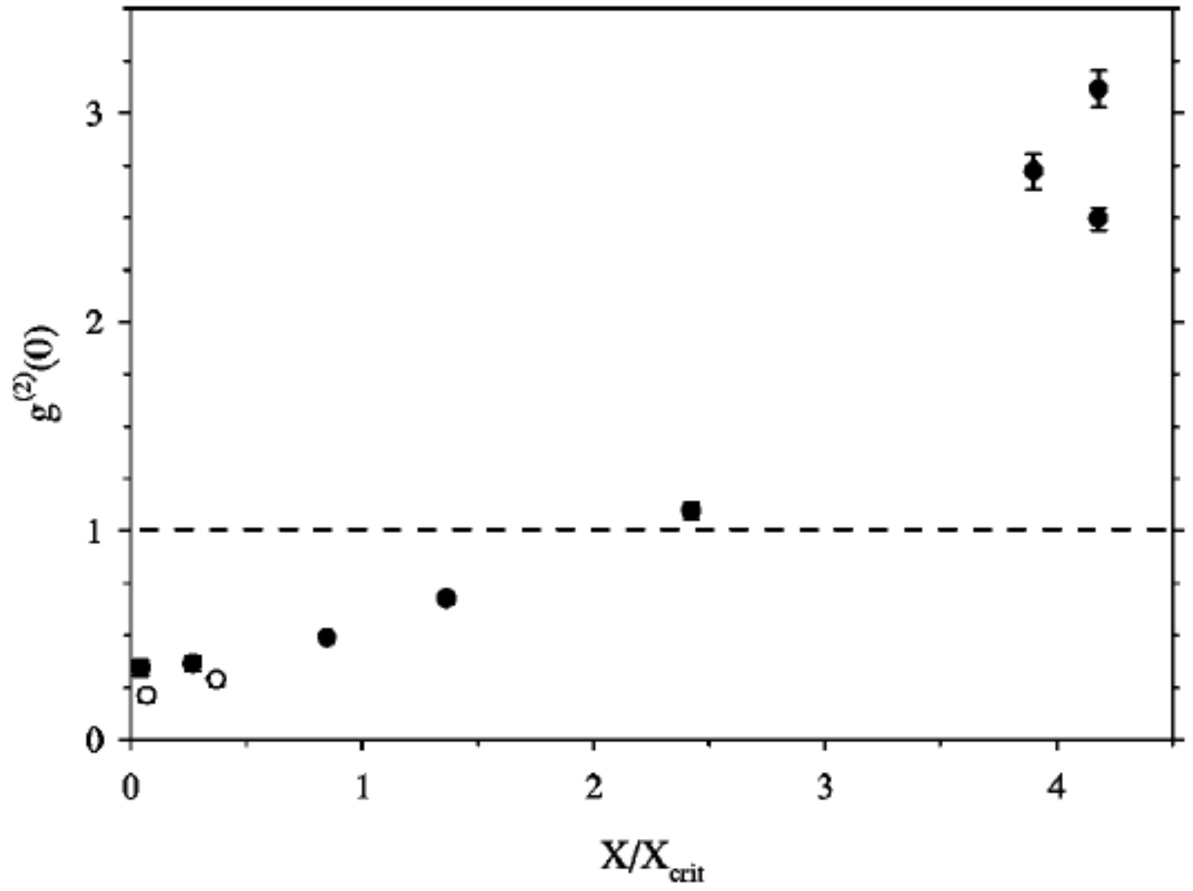
The amplitude of the decaying oscillations is given by

$$\mathcal{A}=-\frac{4C_1^2N}{(1+\gamma_\perp/\kappa)(1+2C_1N)-2C_1}.$$

What happens after a click?



As a function of intracavity intensity



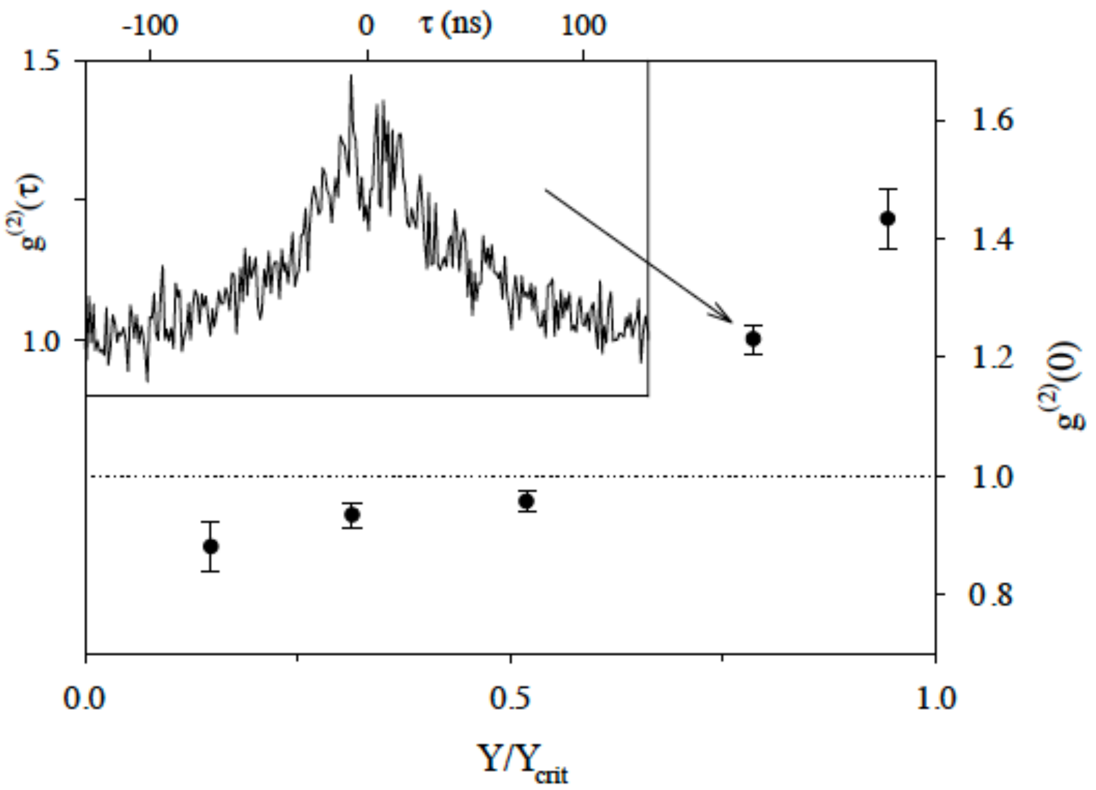
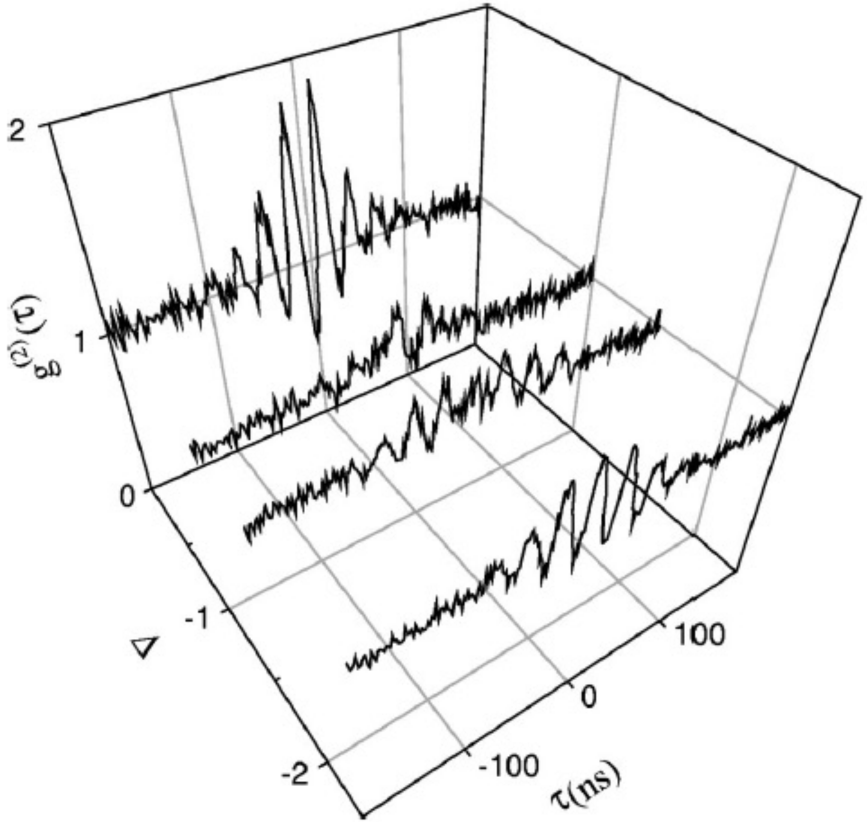
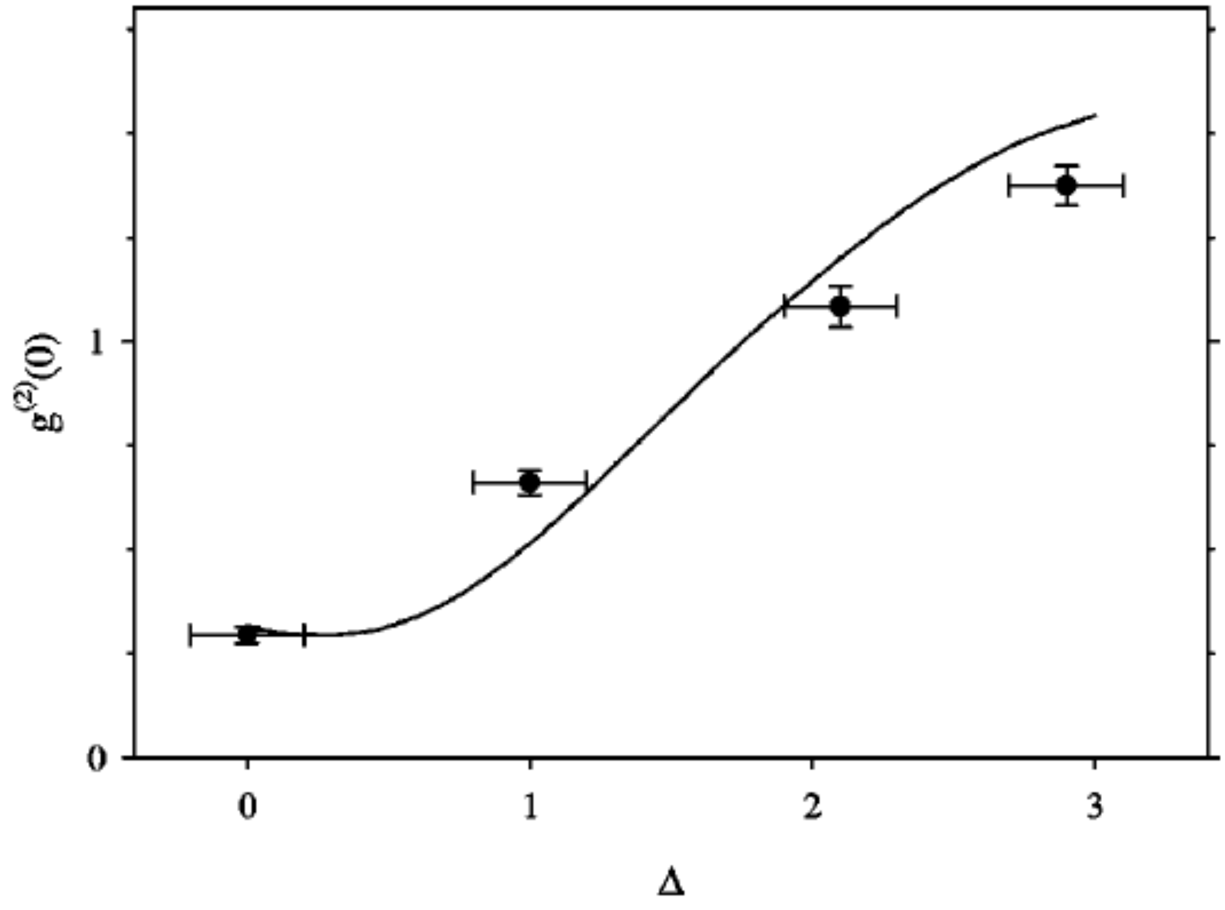


FIG. 3. Evolution of $g^{(2)}(0)$ as the intensity increases ($C_1 = 3.9$, $n_0 = 0.08$, and $N \approx 10$). The statistics change from sub-Poissonian to super-Poissonian. The inset presents the antibunched $g^{(2)}(\tau)$ for the point indicated in the plot.

As a function of detuning



Detuning



effective number of atoms

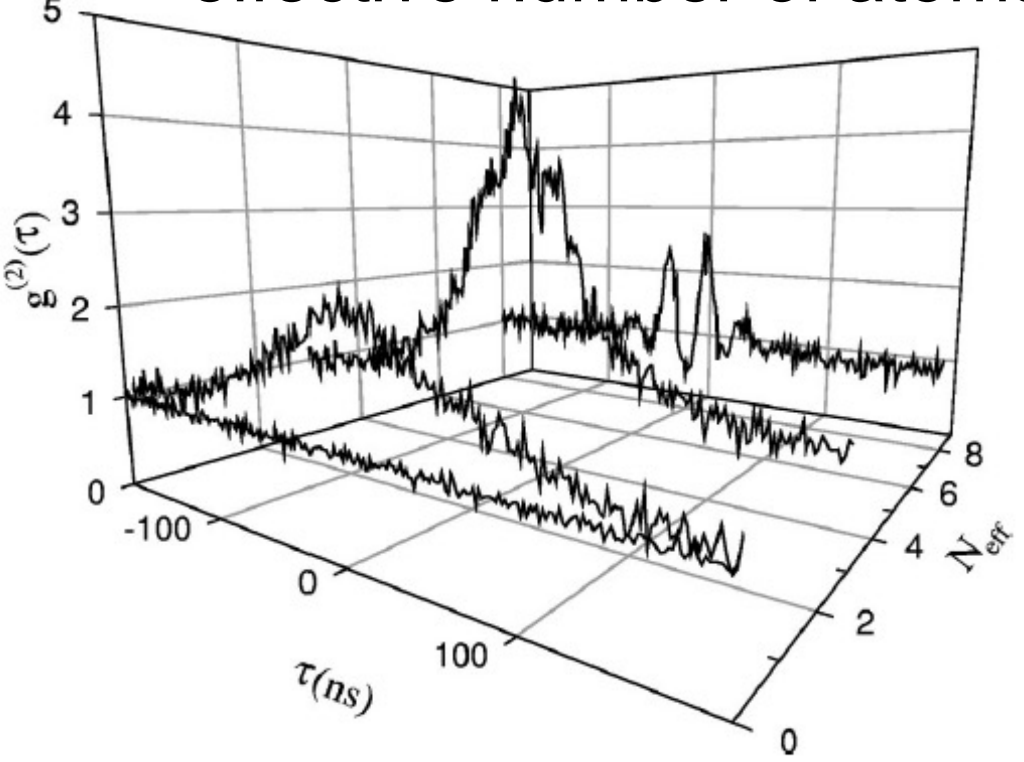


FIG. 11. Evolution of $g^{(2)}(\tau)$ as a function of the effective numner of atoms for $N_{\mathrm{eff}}=0, 0.1, 3$, and 8. Each correlation is taken on resonance. For the cases with atoms, the intracavity intensities

Large background thermal atoms

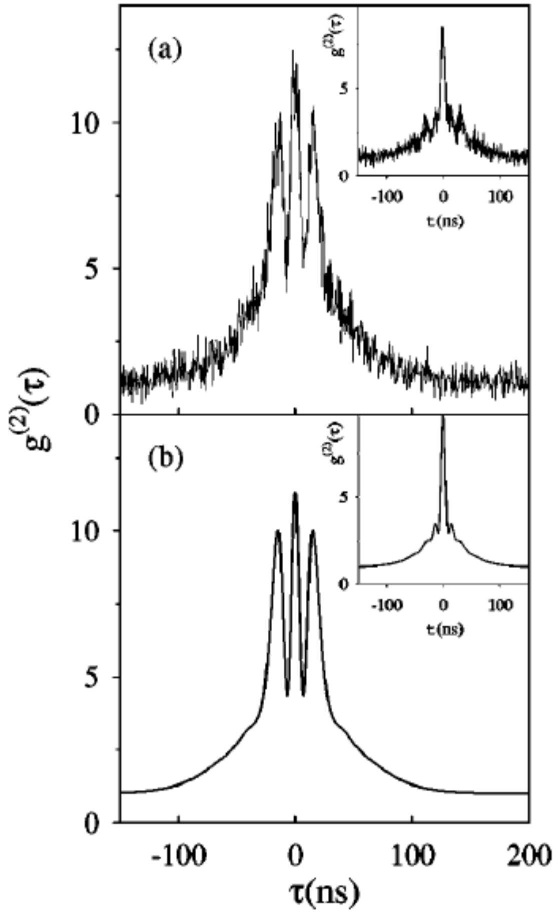


FIG. 10. $g^2(\tau)$ for cavity 3 with a small effective atom number $N \approx 3$, and $X = 0.06$. The main plot (a) shows data collected on resonance. The inset shows data with a detuning of $\Delta = 1.5$. Plot (b)

Bibliography:

H. J. Carmichael, R. J. Brecha, P. R. Rice, “Quantum interference and collapse of the wavefunction in cavity QED,” Optics Communications, **82**, 73 (1992).

S. L. Mielke, G. T. Foster, and L. A. Orozco, “Non-classical intensity correlations in cavity QED,” Phys. Rev. Lett. **80**, 3948 (1998).

G. T. Foster, S. L. Mielke, and L. A. Orozco, “Intensity correlations in cavity QED,” Phys. Rev. A, **61,** 053821 (2000).

Thanks